STAT 2035 Midterm 2 ACE Booklet Solutions

A. The Exponential Distribution

Example 1. Jobs are sent to a printer at an average of 3 jobs per hour.

y= # of jobs sent to the printer per hour   y=0,1,2,...
$y \sim Poisson\ (\lambda = 3 = \mu_y)$

x=time between jobs at the printer in hours   x>0
$x \sim Exponential\ (\lambda = 3)$
E(x)=$\frac{1}{3}$ $which\ means\ a\ job\ arrives\ every\ \frac{1}{3} of\ an\ hour$

a) What is the expected time between jobs?

Solution:

$\lambda = 3$
expected time=E(x)= $\frac{1}{\lambda} = \frac{1}{3}$ hour

b) What is the probability that the next job is sent within 10 minutes?
$\lambda = 3$ per hour

10 min=0.16666 an hour

$\Pr(x \leq a) = 1 - e^{-\lambda a}$

$\Pr(x \leq 10min) = Pr\ (x \leq 0.16666)$hr

$= 1 - e^{-\lambda a}$
$= 1 - e^{-3(0.166666)}$
=0.393

Example 2.

$\mu_x = 100\ 000$ miles

y= # of failures (miles) y=0,1,2,...

$y \sim Poisson\ (\lambda = 100\ 000 = \mu_y)$

x=distance between failures (in # miles) x>0

$x \sim Exponential\ (\lambda = 100\ 000)$

E(x)=100 000 miles

$which\ means\ a\ car\ travels\ 100\ 000\ miles\ on\ average\ before\ failure$

$$\mu_x = \frac{1}{\lambda}$$

$$100\ 000 = \frac{1}{\lambda}$$

$$\lambda = \frac{1}{100\ 000} = 0.00001$$

$$\Pr(x \leq 70\ 000) = 1 - e^{-\lambda b}$$

$$= 1 - e^{-0.00001(70\ 000)}$$
$$= 1 - e^{-0.7}$$
$$= 1 - 0.497$$
=0.503

Example 3.
$\lambda = 3$/hour

y= # of students arriving per hour y=0,1,2,...
$y \sim Poisson\ (\lambda = 3 = \mu_y)$

x=time between students x>0
$x \sim Exponential\ (\lambda = 3)$
E(x)=$\frac{1}{3}$ $which\ means\ a\ student\ arrives\ every\ \frac{1}{3} of\ an\ hour$

a) Prob. more than 10 min before next person arrives

Exponential

Pr(X>10min)=Pr(X>10/60hr)=Pr(X>0.16667)

Pr(X>c)= $e^{-\lambda c}$
Pr(X>0.16667)=$e^{-3(0.1667)} = 0.6065$

b) Prob time between arrivals is between 10 and 20 min?
Exponential Pr(a<X<b)= $e^{-\lambda a} - e^{-\lambda b}$

Pr(10<X<20min)=Pr(10/60<X<20/60 hr)= Pr(0.166667<X<0.3333333)

=$e^{-3(0.1666667)} - e^{-3(0.333333)} = 0.6065 - 0.3679 = 0.239$

Example 5.
$\lambda = 0.6\ downs/hour$

a) Pr(X>c)= $e^{-\lambda c}$
Pr(X>3)= $e^{-0.6(3)} = 0.165$

b) Pr(X$\geq 4/X > 1)$ =Pr(X$\geq$ 3)= $e^{-0.6(3)} = 0.165$

Example 6. $\lambda = 16$/hour

y= # of calls arriving per hour y=0,1,2,...
$y\sim Poisson\ (\lambda = 16 = \mu_y)$

x=time between calls x>0
$x\sim Exponential\ (\lambda = 16)$
E(x)=$\frac{1}{16}$ $which\ means\ a\ call\ arrives\ every\ \frac{1}{16} of\ an\ hour$

Pr $(X < 15/$X>10) is a conditional probability...BUT, remember the MEMORYLESS property of the exponential function (it doesn't remember you have waited 10 minutes. But, it doesn't say Pr(X>15) so it is just finding the probability that the next call occurs in the next 5 minutes, or less than/equal to 5)

Pr$(X \leq a) = 1 - e^{-\lambda a}$
=Pr(X<5min)=Pr(X<5/60 hr))=Pr(X<0.08333)
is in the next 5 min=$1 - e^{-16(0.08333)} = 1 - 0.264 = 0.736$

NOTE: If it just said what is the probability the next call arrives between 10 and 15 min from now....it would be:

Pr(10<X<15min)=Pr(1/6<X<1/4 hr)= $e^{-\lambda a} - e^{-\lambda b}$

= $e^{-16\left(\frac{1}{6}\right)} - e^{-16\left(\frac{1}{4}\right)}$=0.0695-0.0183=0.0512

Example 7.

$$\Pr(x < t) = 0.75$$
$$1 - e^{-\lambda a} = 0.75$$
$$1 - e^{-16a} = 0.75$$
$$0.25 = e^{-16a}$$
$$\ln 0.25 = \ln e^{-16a}$$
$$\ln 0.25 = -16a \ln e$$
$$-1.38629 = -16a$$
$$a = 0.08664\ hour\ \times 60 = 5.2\ minutes.$$

Example 8. We know from unit #1, the median occurs with 50% of the data above it and 50% below it

$\mu = 0.25 \ \ \lambda = \frac{1}{\mu} = \frac{1}{0.25} = 4$

$Pr(x < c) = 0.50$

$1 - e^{-\lambda c}$=0.50

$1 - e^{-4c} = 0.50$

$0.5 = e^{-4c}$

$ln0.5 = lne^{-4c}$

$ln0.5 = -4clne$

$ln0.5 = -4c$

$c = 0.17\ hours\ or\ 0.17 \times 60 = 10.4\ minutes$

Practice Exam Questions on Exponential Distributions

A1. The time in hours required to repair a machine is an exponential random variable with $\lambda = 1/2$.

y= # of repairs per hour   y=0,1,2,...

$y \sim Poisson\ (\lambda = \frac{1}{2} = \mu_y)$

x=time between repairs   x>0

$x \sim Exponential\ (\lambda = 1/2)$

E(x)=$\frac{1}{2}$ $which\ means\ a\ student\ arrives\ every\ \frac{1}{2} of\ an\ hour$

a) What is the probability that the repair time exceeds 3 hours?

$\lambda = \frac{1}{2} = 0.5$

$\mu_x = \frac{1}{\lambda} = 2$

Pr(X>c)= $e^{-\lambda c}$

Pr($x > 3$)=$e^{-0.5(3)} = e^{-1.5}$=0.223

b) What is the conditional probability that a repair takes at least 10 hours, given that its duration exceeds 8 hours?

Pr(X≥10/X>8)= Pr($X \geq 2$) $by\ memoryless\ property$

$= e^{-0.5(2)} = 0.368$

A2. On the average, a certain computer part lasts 12 years. The length of time the computer part lasts is exponentially distributed.

y= # of computer repairs y=0,1,2,...
$y \sim Poisson\ (\lambda = \frac{1}{12} = \mu_y)$

x=# of years before repair is needed x>0
$x \sim Exponential\ (\lambda = \frac{1}{12})$
E(x)=12 $yr\ = \mu\ which\ means\ a\ student\ arrives\ every\ \frac{1}{12} of\ an\ hr$

What is the probability that a computer part lasts more than 6 years?

$\mu = 12$ years
$\lambda = 1/12$ yr
Pr(X>6)=$e^{-\frac{1}{12}(6)} = e^{-0.5} = 0.607$

A3.
Let x be the time between accidents
X$\sim Exponential\ with\ \lambda = 3$

Pr(X>2)=$e^{-3(2)} = e^{-6}$=0.002479

A4. Let x be the time between customers

X$\sim Exponential\ with\ \mu = 10$
$\lambda = 1/10$=0.1

a) Pr(X>20)= $e^{-0.1(20)} = 0.135$

b) Pr(X≥15/X>12)= Pr$(X \geq 3)\ by\ Memoryless\ property = e^{-0.1(3)}$=0.7408

*A5. $mean = 25, so\ \lambda = 1/25$

a) Pr(X>35)= $e^{-\frac{1}{25}(35)} = 0.247$

b)Pr(spend more than 30 minutes/ still in store after 25 minutes)

=Pr(X>5 minutes)

=$e^{-1/25(5)} = 0.819$

B. Continuous Random Variables

Example 1.
Pr(9<x<15)= Area rectangle = (length)(width)=(6)(1/10)=0.60

Example 2.

Graph f(x)=x/50 for $0 \leq x \leq 10$, where f(x)=0 otherwise

Pr(X<6)= Area triangle= $\frac{bxh}{2} = \frac{(6)(0.12)}{2} = 0.36$

Example 4.

From the prep book, for uniform distributions,
$\mu_x = \frac{a+b}{2}$ and $\sigma_x = \frac{b-a}{\sqrt{12}}$
and we know the mean squared and the standard deviation are equal in this question and
From (0,b) , we know that a=0.

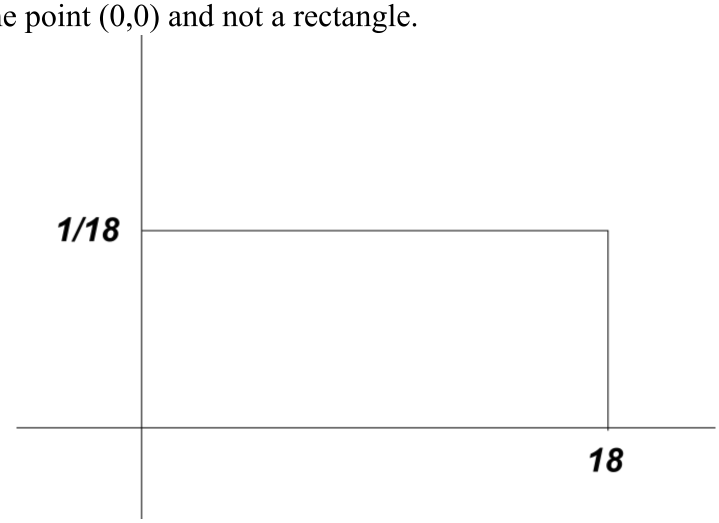
$$\left(\frac{b}{2}\right)^2 = \frac{b}{\sqrt{12}}$$
$$\frac{b^2}{4} = \frac{b}{\sqrt{12}}$$
$$\sqrt{12}b^2 - 4b = 0$$
$$b\,(\sqrt{12}b - 4) = 0$$
$$\sqrt{12}b - 4 = 0$$
$$b = \frac{4}{\sqrt{12}} = \frac{4}{\sqrt{4}\sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$$

Note: b is not 0 as then it would just be the point (0,0) and not a rectangle.

Example 5.

Area=1

(18)(h)=1



h=1/18

Pr(wait at least 12 min)= lengthxwidth

=(18-12)(1/18) = 6/18 = 0.33

Example 6.

To find the value of a, find the area of each of the shapes and set it equal to 1, since the total area is the same as the total probability

A=$(2)\left(\frac{1}{2}a\right) + (6)\frac{1}{2}a = 1$

$$1 = \frac{1}{2}a + 3a$$

$$1 = \frac{7}{2}a$$

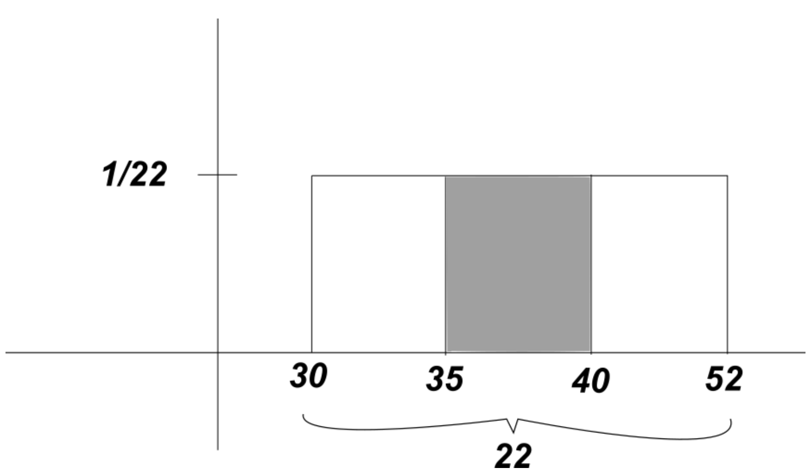
$$a = \frac{2}{7}$$

Example 7.

A = less than 40

B = more than 35

$$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{\frac{5}{22}}{\left(\frac{52-35}{22}\right)} = \frac{5}{17} = 0.294$$



Practice Exam Questions on Random Variables

B1. The probability density function f(x) of a continuous random variable X is defined by f(x)=kx if 0<x<7 and f(x)=0 otherwise. Find the value of k.

The total area under the graph must be equal to 1.

bh/2 = 1

(7)(7k)/2 = 1
49k/2=1
49k=2
k=2/49

B2. Draw a graph and find the area of the rectangle between x=4 and x=7

Area=lxw= (7-4)(1/10)=3/10=0.30

Therefore, the prob. he received between 4 and 7 calls is 0.3

B3. uniform between (1.4, 1.6)

a) E(X)= $\frac{a+b}{2} = \frac{1.4+1.6}{2} = 1.5$

If you draw a rectangle with width on the x-axis from 1.4to 1.6...this is =0.2 units...so the length of the rectangle is 1/0.2=5

b) Expected number with voltage more than 1.5volts

=Pr(more than 1.5 volts) x 120 batteries= lengthxwidthx120

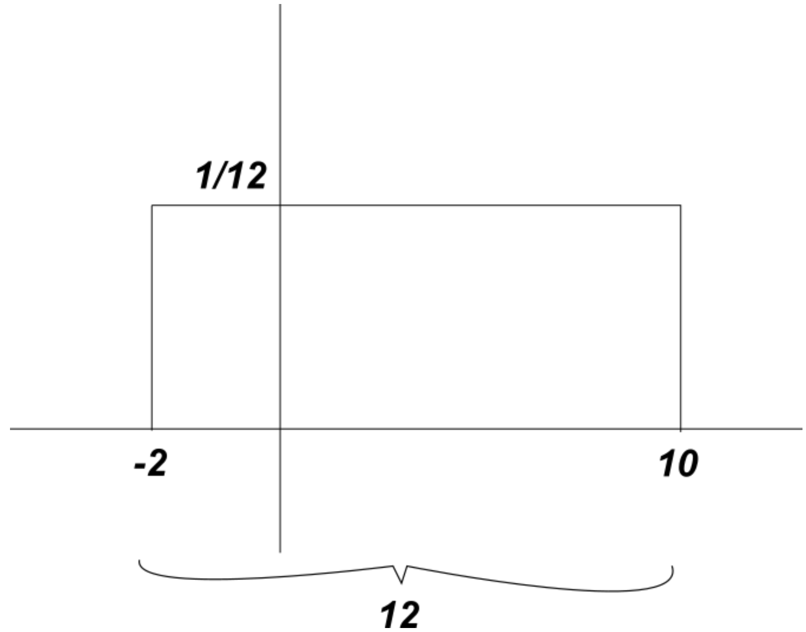
=(1.6-1.5)(5)(120) =60 batteries

B4.

$0.25 \times 12 = 3$ (from -2)

$-2 + 3 = 1$

∴ 1 is the lower quartile



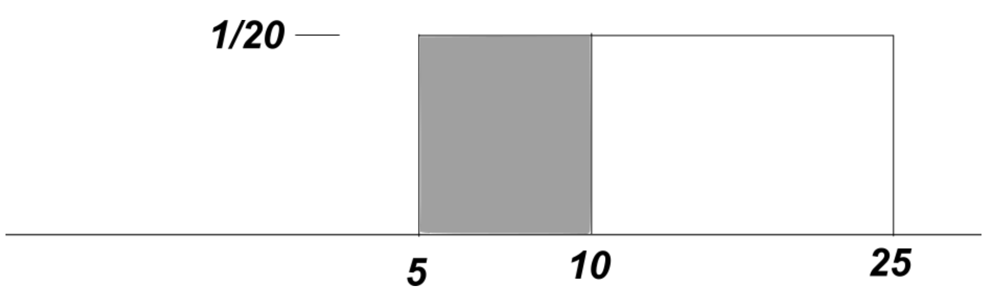
B5. Area of a rectangle is length x width

$$\Pr(x > 0 / x < 4) = \frac{\Pr(x > 0 \cap x < 4)}{\Pr(x < 4)}$$

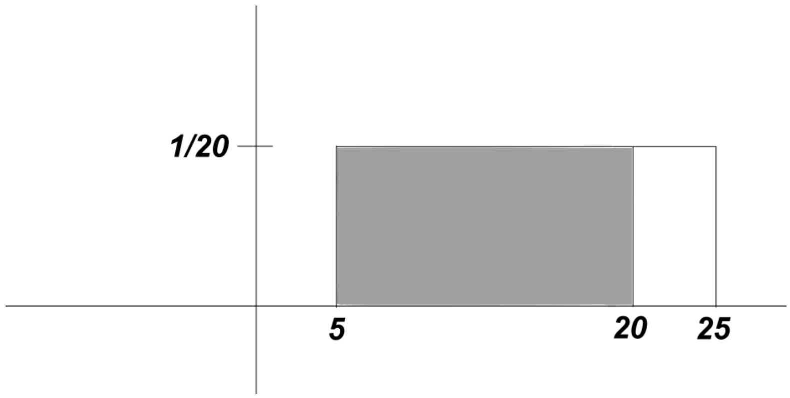
$$= \frac{4 \times \frac{1}{12}}{6 \times \frac{1}{12}}$$

$$= \frac{4}{6} = \frac{2}{3}$$

B6.



= $\Pr(x < 10) = 5 \times \frac{1}{20} = \frac{5}{20} = 0.25$

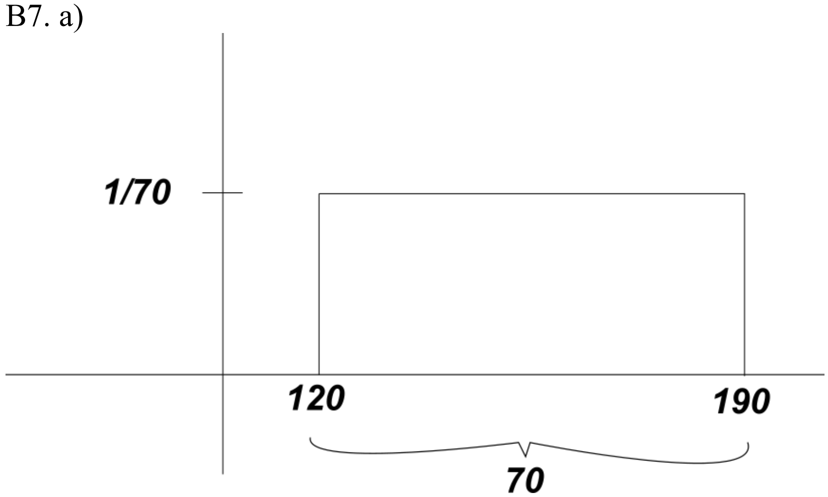


$\Pr(x < 20) = 15 \times \frac{1}{20} = \frac{15}{20} = 0.75$

$$\Pr\,(x < 10/\,x < 20) = \frac{\Pr\,(x<10 \cap x<20)}{\Pr\,(x<20)} = \frac{\Pr\,(x<10)}{\Pr\,(x<20)} = \frac{0.25}{0.75} = \;\; 0.33$$

The answer is C.

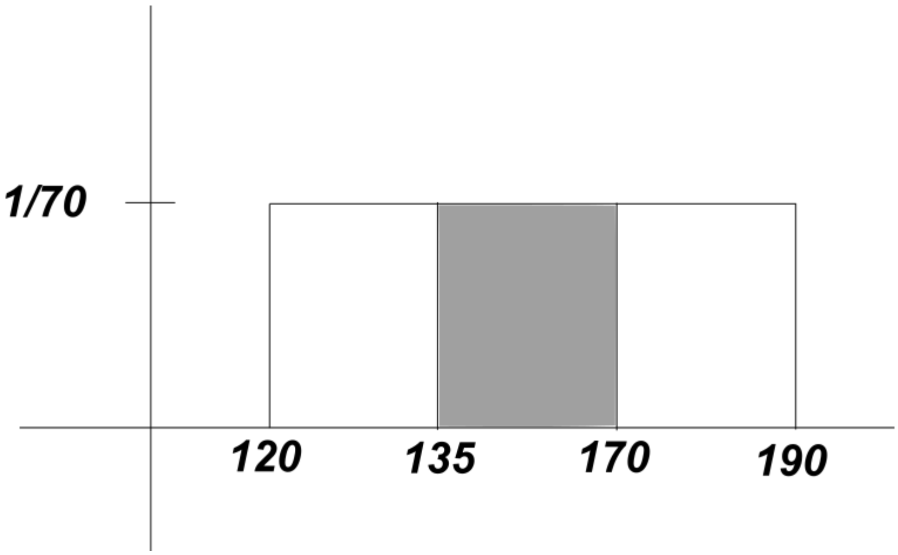
B7. a)



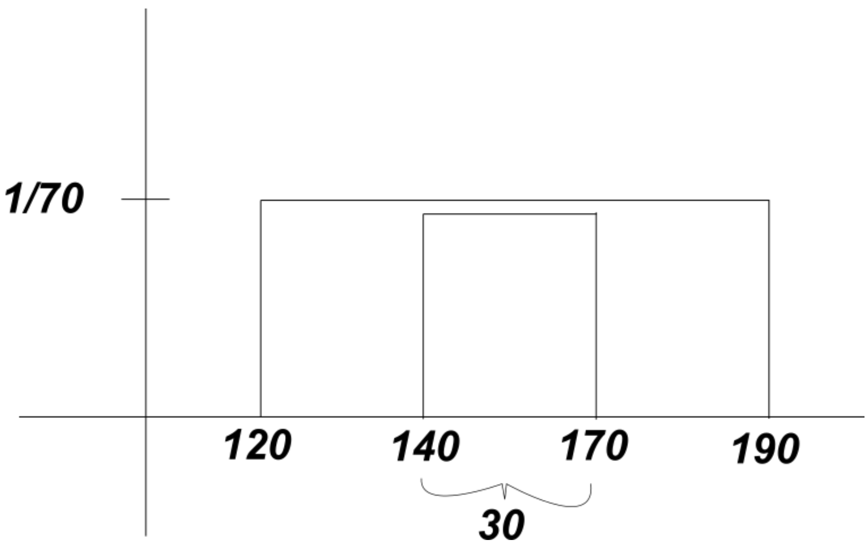
$0.85 \times 0.70 = 59.5$

$\therefore 120 + 59.5 = 179.5$

b)$\Pr(A) = 50 \times \frac{1}{70} = \frac{50}{70} = 0.714$



$\Pr(B) = 35 \times \frac{1}{70} = 0.50$



$\Pr(A \cap B) = \frac{30}{70} = 0.429$

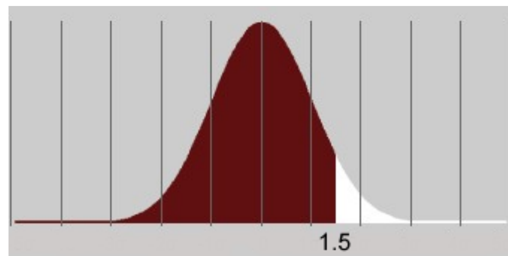
$$\begin{aligned}\Pr(A \cup B) &= \Pr(A) + \Pr(B) - \Pr(A \cap B)\\ &= 0.714 + 0.50 - 0.429\\ &= 0.785\end{aligned}$$

C. Normal Distribution

<u>Example</u> On the left from 8 to 12 is 1/2(95%)=47.5% and on the right from 12 to 14 there is 1/2(68%)=34%, so the total is 81.5%

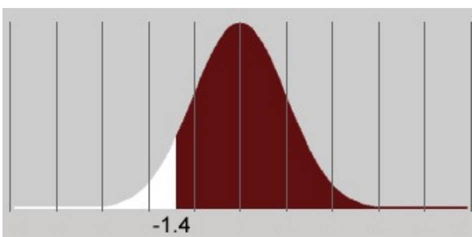
<u>Example 1</u>. Find each of the following probabilities by using the table for Z.

a) Pr[Z<1.5]



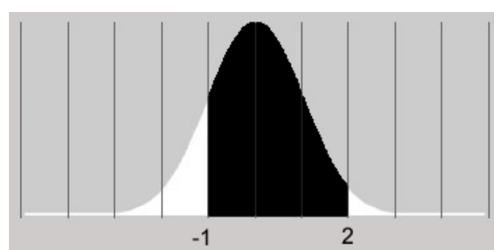
Therefore, the answer is Pr[Z<1.5]= 0.9332

b) Pr[Z>-1.4]



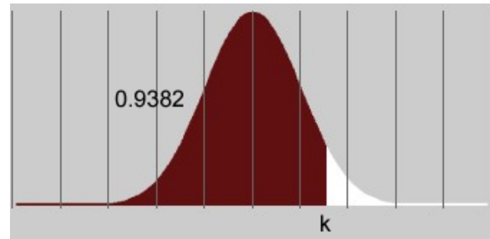
Therefore, the answer is Pr[Z>-1.4] = 1- Pr[Z< -1.4] = 1 – 0.0808= 0.9192

c) Pr[-1<Z<2]



Pr[Z<2] - Pr[Z<-1] = 0.9772 - 0.1587 = 0.8185

Example 2. Find k such that Pr(Z<k)=0.9382

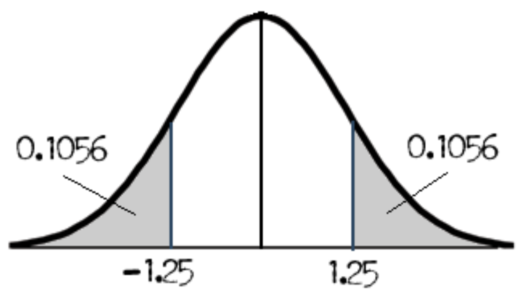


Look up the area 0.9382 and find "k" along the left side of the table.

Therefore, k=1.54

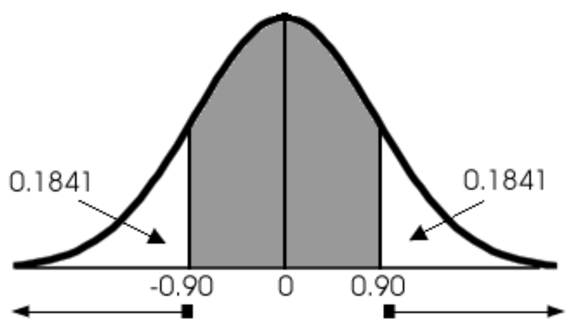
Example 3. Pr(Z<k)=1-0.7=0.3
Look up the area 0.3 and you get k= - 0.525

Example 4.
$\Pr(|Z| > 1.25) = 2(0.1056) = 0.2112$



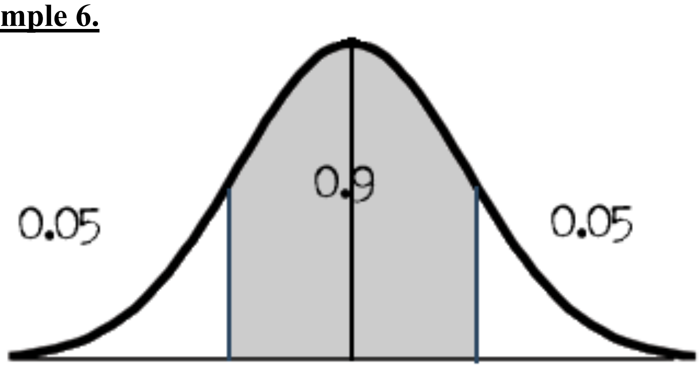
Example 5.

Find Z. Draw the z curve with labeling -0.90 and +0.90 and shade between them. Look up Pr(Z<0.90)-P(Z<-0.90)=0.6318
Or do 1- (0.1841)x2=0.6318



Look up the area 0.9+0.05=0.95 in the body of the table...Z=±1.645

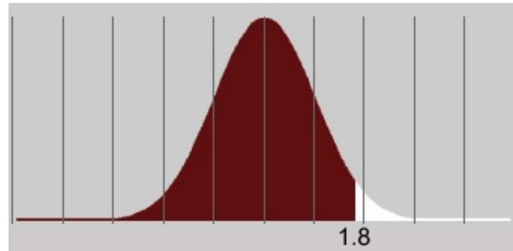
Example 6.



Look up Area 0.95 in the body (or look up 0.05) and get 1.645, so k=Z=1.645

Practice Exam Questions on Normal Distributions

C1. For the standard normal random variable Z, find the value of Pr[Z<1.6].



A. 0.0548	B. 0.9452	C. 0.8554	D. 0.1446	E. None of the above

Pr (Z<1.6)=0.9452
The answer is b).

C2. For the standard normal random variable Z, find the value of Pr[Z>-0.80].

A.0.7881	B. 0.80	C. 0.2119	D. 0.5319	E. None of the above

Pr(Z> - 0.80) = Pr(Z<0.80) = 0.7881 or do 1- Pr(Z<-0.8)= 1- 0.2119=0.7881
The answer is a).

C3. For the standard normal random variable Z, find the value of Pr[-1.20<Z<1.20].

A. 2.4	B. 0.9918	C. 0.1151	D. 0.7698	E. None of the above

Pr (-1.20<Z<1.20)= Pr(Z<1.2) - Pr(Z<-1.2)
=0.8849 – 0.1151
=0.7698
The answer is d).

C4. For the standard normal random variable Z, what is the value of Pr[Z>1.76]?

A.0.0392	B. 0.9608	C. 0.9554	D. 0.0446	E. None of the above

Pr (Z> 1.76)= 1 - Pr(Z< 1.76)
= 1 - 0.9608
=0.0392
The answer is a).

C5. If Z is the standard normal random variable, find Pr[-1 < Z < 1].

A. 0.0228	B. 0.9772	C. 0.1587	D. 0.8413	E. None of the above

Pr (-1<Z<1)= Pr(Z<1) - Pr(Z<-1)
=0.8413 – 0.1587
=0.6826
The answer is e).

C6. Find the value of k if it is known that Pr[k<Z<1.5]= 0.0483 , where Z is the standard normal random variable.

A. 1.2	B. -1.2	C. 0.8849	D. 1.66	E. none of the above

Pr(Z<1.5)= 0.9332
0.9332 – 0.0483=0.8849
Look up area 0.8849 and you get k=1.2

The answer is a).

C7. Use the table for the standard normal random variable Z to find Pr[-0.65<Z<1.92].

A. 0.6226	B. 0.2284	C. 0.7148	D. 0.2852	E. None of the above

Pr (-0.65<Z<1.92)=Pr(Z<1.92) - Pr(Z<-0.65)
Pr(Z<1.92) - 0.2578
=0.9726 -0.2578
=0.7148

The answer is c).

C8. Use the table for the standard normal random variable Z to find a value of k for which Pr[Z<k]= 0.9495

A. 0.9495	B. 0.8264	C. -1.64	D. 1.64	E. None of the above

Pr(Z<k)=0.9495

Find the area=0.9495 by looking in the body of the chart...we get k=1.64
The answer is d).

Normal Random Variables

Example 7.

$$Z = \frac{X - \mu}{\sigma}$$

Pr(X<14)=Pr (Z<$\frac{14-8}{3}$) $= \Pr\,(Z < 2)$
=0.9772

Example 8.

$$Z = \frac{X - \mu}{\sigma}$$

mean= μ=5

$\sigma = \sqrt{9} = 3$

$$\Pr(\mathrm{X} > 10) = \Pr\left(Z > \frac{10-5}{3}\right) = \Pr\left(Z > \frac{5}{3}\right) = \Pr(Z > 1.67)$$
$= 1 - 0.9525 = 0.0475$

Example 9.

$$Z = \frac{X - \mu}{\sigma}$$

mean= μ=11

$\sigma = 2$

$$= \Pr\,(\frac{14-11}{2} < Z < \frac{15-11}{2})$$
$= \Pr(1.5 < Z < 2)$
=Pr(Z<2) - Pr(Z<1.5)
=0.9772 -0.9332
=0.044

Example 10. X is a normal random variable with unknown mean μ and standard deviation σ=3. If Pr[X<25]=0.9772, what is the value of μ?

Look up the area 0.9772 in the body and you get Z=2.

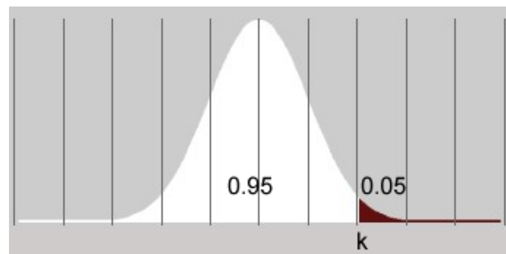
$$Z = \frac{X-\mu}{\sigma}$$

$$2 = \frac{25 - \mu}{3}$$

μ =19

Example 11. ...in the top 5% of all students taking this exam?

$\mu = 60$
$\sigma = 10$



Pr(Z<k)=0.95 9(we need the area below the line)
Look up the area 0.95 on the table and k=1.645

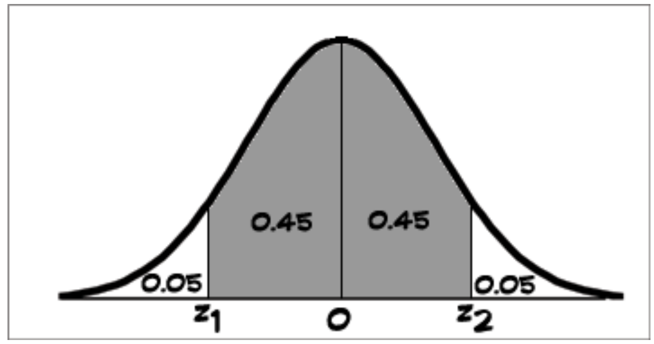
$$Z = \frac{X-\mu}{\sigma}$$

$$1.645 = \frac{X-60}{10}$$

X=76.5

Therefore, a student must score 76.5

***Example 12**.
$\mu = 75$
$\sigma^2 = 16$
$\sigma = 43$
$Z = \frac{X - \mu}{\sigma}$

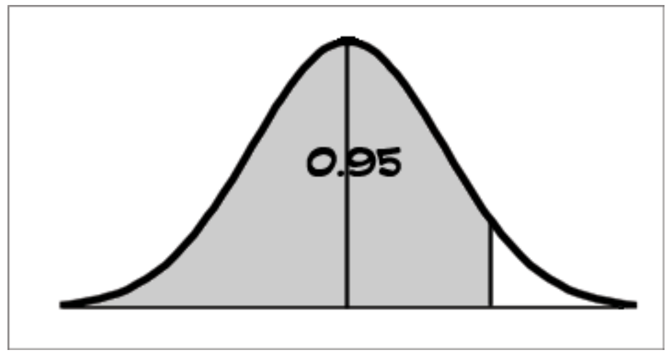


Look up area 0.95 to find Z_2=1.645

Look up area 0.05 to find Z_1= -1.645
$-1.645 = \frac{X - 75}{4}$
x_1=68.4

$1.645 = \frac{X - 75}{4}$
x_2=81.6

The interval is from (68.4, 81.6)



Example 13.

$\mu = 19.75oz$
$\sigma = 0.5$

Pr(X<c)=0.70 find c

Area=0.70 Look this up in the BODY of the Z table and we get Z=0.525

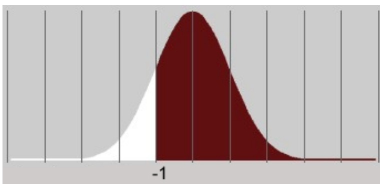
$Z = \frac{X - \mu}{\sigma}$
$0.525 = \frac{X - 19.75}{0.5}$

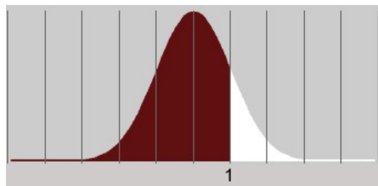
x=c=20.01oz

Practice Exam Questions on Normal Distributions

C9. X is a normal random variable with mean μ=90 and standard deviation σ=10. Find Pr[X>80].

A. 0.8413	B. 0.1587	C. 0.9222	D. 0.0778	E. None of the above





$$Z = \frac{80-90}{10} = -1$$

Pr(X>80)=Pr(Z>-1)= 1-Pr(Z<-1)=1-0.1587=0.8413...the answer is a).

C10. X is a normal random variable with mean μ=8 and standard deviation σ=2. Find Pr[12<X<14].

A. 1	B. 0.9987	C. 0.9772	D. 0.8413	E. None of the above

Pr(12<X<14)=Pr $\left(\frac{12-8}{2} < Z < \frac{14-8}{2}\right)$
=Pr(2<Z<3)
=Pr(Z<3) - Pr(Z<2)
=0.9987 - 0.9772
=0.0215
The answer is e).

C11. Pr(X<520)=Pr(Z<$\frac{520-500}{20}$)=Pr(Z<1)= 0.8413

C12. X is a normal random variable with mean 35 and standard deviation 5.

Pr(30<X<40)=Pr$\left(\frac{30-35}{5} < Z < \frac{40-35}{5}\right) = \Pr(-1 < Z < 1)$

=Pr(Z<1) - Pr(Z<-1)

=0.8413 – 0.1587=0.6826

C13. ...greater than 630?

Let *X* be the test score. Then $X \sim \mathrm{N}(\mu,\sigma)$

with $\mu = 500$, $\sigma = 100$. So

$$\Pr(X > 630) = \Pr\left(Z = \frac{X-\mu}{\sigma} > \frac{630-500}{100}\right)$$

$$= \Pr(Z > 1.3) = 1 - \Pr(Z < 1.3) = 1 - 0.9032 = 0.0968.$$

C14. ... and standard deviation 10.

(a) What percentage of the children have IQ's greater than 125?

Let *X* be the child's IQ. Then $X \sim \mathrm{N}(\mu,\sigma)$

with $\mu = 100, \sigma = 10$. So

$$\Pr(X > 125) = \Pr\left(Z = \frac{X-\mu}{\sigma} > \frac{125-100}{10}\right)$$

$$= \Pr(Z > 2.5) = 1 - \Pr(Z < 2.5) = 1 - 0.9938 = 0.0062$$

(b) About 90% of the children have IQ's greater than what value?

Look up the area 0.10 in the body (since the area above your line is 90% of the graph shaded, so the area below the line will be 10%) and get Z= -1.28

$$\Pr(Z > z) = 0.90 \quad \Rightarrow \quad z = -1.28.$$

$$z = \frac{x-\mu}{\sigma} \quad \Rightarrow \quad x = \mu + z\sigma = \mu - 1.28\sigma$$

So $x = 100 - 1.28 \cdot (10) = 87.2.$

Thus, 90% of the children have IQ's greater than 87.2.

C15. (a) What is the probability of getting a 81 or less on the exam?

Let *X* be the final grade. Then $X \sim N(\mu, \sigma)$ with $\mu = 73$, $\sigma = 8$. Then

$$\Pr(X \le 81) = \Pr\left(Z = \frac{X-\mu}{\sigma} \le \frac{81-73}{8}\right) = \Pr(Z < 1) = 0.8413.$$

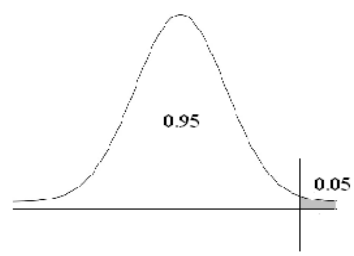
(b) What percentage of students scored between 65 and 89?

$$\Pr(65 < X < 89) = \Pr\left(\frac{65-73}{8} < Z < \frac{89-73}{8}\right) = \Pr(-1 < Z < 2)$$

$$= \Pr(Z < 2) - \Pr(Z < -1) = 0.9772 - 0.1587 = 0.8185.$$

c) Only 5% of the students taking the test scored higher than what grade?

Look up area=0.95 (below the line) in the body of the chart and get Z=1.645



$$\Pr(Z > z) = 0.05 \Rightarrow z = 1.645.$$

$$z = \frac{x-\mu}{\sigma} \Rightarrow x = \mu + z\sigma = \mu + 1.645\sigma$$

So $x = 73 + 1.645 \cdot (8) = 86.16$.

Therefore, 5% of the students scored higher than 86%.

C16.

$$Z = \frac{X-\mu}{\sigma}$$

Pr(X<41)=Pr $\left(Z < \frac{41-45}{5}\right) = \Pr(Z < -0.8) = 0.2119$

C17.

$$Z = \frac{X-\mu}{\sigma}$$

Pr(9<Z<10)=Pr $\left(\frac{9-8}{2} < Z < \frac{10-8}{2}\right) = (0.5 < Z < 1)$

=Pr(Z<1) - Pr(Z<0.5)

=0.8413 – 0.6915 =0.1498

C18. ... earn less than \$3400 per month?

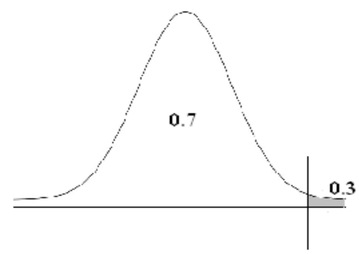
$\mu = 3400$
$\sigma = 200$

$$Z = \frac{X - \mu}{\sigma}$$

Pr(X<3400)=Pr(Z<$\frac{3400-3600}{200}$) $= \Pr(Z < -1) = 0.1587$

C19. Let X be the waiting time (in days). Then

$X \sim N(\mu, \sigma)$ with $\mu = 120,\ \sigma = 20.$



Look up the area 0.70 in the body of the table (area below your line) and get Z=0.52

So $\Pr(Z > z) = 0.30 \Rightarrow z = 0.52.$

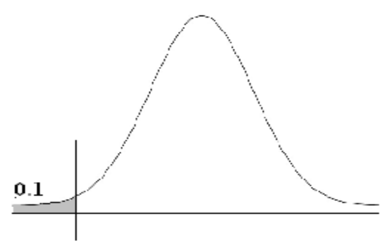
$$z = \frac{x-\mu}{\sigma} \Rightarrow x = \mu + z\sigma = \mu + 0.52\sigma$$

$$= 120 + 0.52 \cdot (20) = 130.4.$$

Therefore, 30% of the patients must wait for more than 130 days for a heart transplant.

(b) What is the longest time spent waiting for a heart transplant that would still place a patient in the bottom 10% of waiting times?

Look up area 0.10 in the body! We can't as it isn't on our chart. But, we know from our picture that Z is negative.



So, look up the area 0.90 and get Z=1.28 and by symmetry this is the same area, but on the opposite side. So, our Z is -1.28.

$\Pr(Z < z) = 0.10 \Rightarrow z = -1.28.$

$$z = \frac{x-\mu}{\sigma} \Rightarrow x = \mu + z\sigma = \mu - 1.28\sigma$$

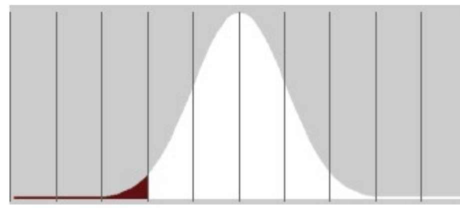
$$= 120 - 1.28 \cdot (20) = 94.4.$$

Therefore, 10% of the patients have to wait for less than 94 days for a heart transplant.

D. Normal Approximation to the Binomial Distribution

Example 1.
Pr(X=4) use approximation Pr(3.5<Y<4.5)
Pr(X<5)=Pr(X≤ 4) $= \Pr(Y < 4.5)$

Pr(X≤ 7) $= \Pr(Y < 7.5)$
Pr(X>8)=Pr(X≥ 9) $= \Pr(Y > 8.5)$



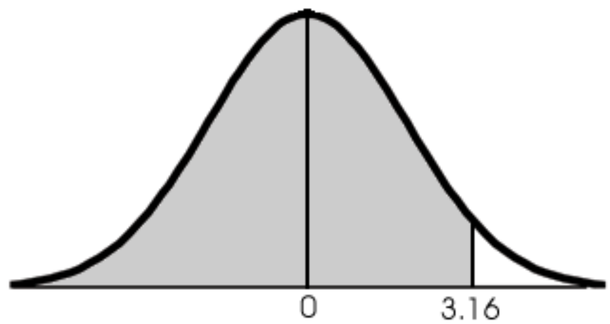
Example 2.

$np = 200(0.1) \geq 5$ √
$nq = 200(0.9) \geq 5$ √
$\therefore$ *use normal approximation*
$\mu = np = 200(0.1) = 20$
$\sigma = \sqrt{npq} = \sqrt{20(0.9)} = 4.24$

Use the number line to find Y

$\Pr(X \geq 35) = \Pr(Y > 34.5)$

$Z = \frac{Y - \mu}{\sigma}$



$= \Pr(Z > \frac{34.5-20}{4.24})$
$= \Pr(Z > 3.42)$
$= 1 - \Pr(Z<3.42)$
$= 1 - 0.9997$
$= 0.0003$

Example 3.

$np = 100\left(\frac{1}{6}\right) \geq 5$ √
$nq = 100\left(\frac{5}{6}\right) \geq 5$ √
∴use normal approximation
$\mu = np = 100\left(\frac{1}{6}\right) = 16.\overline{6}$
$\sigma = \sqrt{npq} = \sqrt{16.6(\frac{5}{6})} = 3.73$
$n = 100$
$\Pr(X = 24) = \Pr(23.5 < Y < 24.5)$ using the number line

$Z = \frac{Y - \mu}{\sigma}$

$= Pr\left(\frac{23.5-16.\overline{6}}{3.73} < Z < \frac{24.5-16.\overline{6}}{3.73}\right)$

$= \Pr(1.83 < Z < 2.10)$
$= \Pr(Z < 2.10) - \Pr(Z < 1.83)$
$= 0..9821 - 0.9664$
$= 0.0157$

Example 4.

$\Pr(X \geq 18) = \Pr(9.5 < Y < 15.5)$ Use the number line to find Y

$$Z = \frac{Y - \mu}{\sigma}$$

$= \Pr(\frac{9.5-16.\overline{6}}{3.73} < Z < \frac{15.5-16.\overline{6}}{3.73})$
$= \Pr(-1.92 < Z < -0.31)$
$= \Pr(Z < -0.31) - \Pr(Z < -1.92)$
=0.3783-0.0274
$= 0.3509$

Example 5.

np=60(0.25)=15
nq=60(0.75)=45
both greater or equal to 5

n=60, p=0.25 and q=0.75
$\mu = np = 60(0.25) = 15 \; and \; \sigma = \sqrt{npq} = \sqrt{60(0.25)(0.75)} = 3.354$
$\Pr(X \geq 18) = \Pr(Y > 17.5)$ Use the number line to find Y

$$Z = \frac{Y - \mu}{\sigma}$$

$$\Pr\left(Z > \frac{17.5 - 15}{3.354}\right)$$

=Pr(Z>0.75)

=1- 0.7734
=0.2266

Practice Exam Questions on Approximation to the Normal Distribution

*D1. n=100 check np=50≥ 5 $and\ nq = 50 \geq 5$, so we can approximate

p=0.5

q=0.5

$\mu = np = 100(0.5) = 50$
$\sigma = \sqrt{npq} = \sqrt{50(0.5)} = 5$

Pr(X>65)=Pr(Y>65.5) Use the number line to find Y

$Z = \frac{Y - \mu}{\sigma}$

=Pr$\left(Z > \frac{65.5-50}{5}\right) = \Pr(Z > 3.1) = 1 - \Pr(Z < 3.1) = 1 - 0.999 = 0.001$

*D2. $\mu = 100, \sigma = 10$

Pr(X$\leq$ 115) $= \Pr(Y < 115.5)$ Use the number line to find Y

$Z = \frac{Y - \mu}{\sigma}$
$= \Pr\left(Z < \frac{115.5 - 100}{10}\right) = \Pr(Z < 1.55) = 0.9394$

*D3. a) check np=30≥ 5 $and\ nq = 180(\frac{5}{6}) \geq 5$, so we can approximate

$\mu = np = 180(1/6) = 30$

b) $\sigma = \sqrt{npq} = \sqrt{30(5/6)} = 5$

c) $\Pr(X \leq 15) = \Pr(Y < 15.5)$ Use the number line to find Y

$$Z = \frac{Y - \mu}{\sigma}$$

$$= \Pr\left(Z < \frac{15.5 - 30}{5}\right) = \Pr(Z < -14.5/5) = \Pr(Z < -2.9)$$

=Pr(Z>2.9)
=1-Pr(Z<2.9)
=1-0.9981=0.0019

*D4.
a) check np=50≥ 5 $and\ nq = 50 \geq 5$, so we can approximate

$\mu = np = 100(0.5) = 50$

b) $\sigma = \sqrt{npq} = \sqrt{50(0.5)} = 5$

c) $\Pr(\mathrm{X} \geq 45) = \Pr(\mathrm{Y} > 44.5)$ Use the number line to find Y

$$Z = \frac{Y - \mu}{\sigma}$$

$$= \Pr\left(\mathrm{Z} > \frac{44.5 - 50}{5}\right) = \Pr(\mathrm{Z} > -1.1) = 1 - 0.1357 = 0.8643$$

D5. $x = \#\ tails$
$n = 18$
$q = heads = \frac{2}{3}$
$p = tails = \frac{1}{3}$ (p must be tails as X=# of tails)

check np=6$\geq$ 5 $and\ nq = 18\left(\frac{2}{3}\right) = 12 \geq 5$, so we can approximate

a) $\mu = np$
$= 18\left(\frac{1}{3}\right) = 6$

b)

$\sigma = \sqrt{npq}$
$= \sqrt{6\left(\frac{2}{3}\right)} = \sqrt{4} = 2$

c)
$Z = \frac{Y - \mu}{\sigma} = \frac{12.5 - 6}{2} = 3.25$

$Pr(X \leq 12)$
$= Pr(Y < 12.5)$
$= Pr(Z < 3.25)$
$= 0.9994$

D6.

$p = get\ red = 0.5$
$q = 0.5$

check np=50≥ 5 $and\ nq = 50 \geq 5$, so we can approximate

a) $= np$
$= 100(0.5) = 50$

b) $= \sqrt{npq}$
$= \sqrt{50(0.5)} = 5$

c)
$Z_1 = \frac{41.5 - 50}{5} = -1.7$
$Z_2 = \frac{56.5-50}{5}$ =1.3

$Pr(42 \leq X \leq 56)$
$= Pr(41.5 < Y < 56.5)$Use the number line to find Y
$= Pr(-1.7 < Z < 1.3)$
$= 0.9032 - 0.0446$

=0.8586

D7. $n = 100$
$p = q = 0.5$
check np=50≥ 5 $and\ nq = 50 \geq 5$, so we can approximate

$\mu = np = 50$
$\sigma = \sqrt{npq} = \sqrt{25} = 5$

$Pr(X \leq 55)$
$= Pr(Y < 55.5)$ Use the number line to find Y

$$Z = \frac{Y - \mu}{\sigma} = \frac{55.5 - 50}{5} = \frac{5.5}{5} = 1.1$$

$\therefore Pr(Z < 1.1) = 0.8643$

D8. n=100

p=0.5

q=0.5

$\mu = np = 100(0.5) = 50$
$\sigma = \sqrt{npq} = \sqrt{50(0.5)} = 5$

Pr(X>60)=Pr(Y>60.5) Use the number line to find Y
$Z = \frac{Y-\mu}{\sigma}$

$=\Pr\left(Z > \frac{60.5-50}{5}\right) = \Pr(Z > 2.1)$

$= 1 - \Pr(Z < 2.1) = 1 - 0.9821 = 0.0179$

*D9. $\mu = 100$

$\sigma = 10$

Pr(X$\leq 110) = \Pr(Y < 110.5)$ Use the number line to find Y

$Z = \frac{Y-\mu}{\sigma}$

$= \Pr\left(Z < \frac{110.5-100}{10}\right)$

$= \Pr(Z < 1.05) = 0.8531$

*D10. a) Find the expected value of X.

$\mu = np = 100(0.5) = 50$
b) Find the standard deviation of X.

$\mu = np = 100(0.5) = 50$
$\sigma = \sqrt{npq} = \sqrt{50(0.5)} = 5$

c) Find an approximation for the probability that Stacey turns up no more than 50 red cards.

$\Pr(X \leq 50) = \Pr(Y < 50.5)$ Use the number line to find Y

$$Z = \frac{Y - \mu}{\sigma}$$

$\Pr\left(Z < \frac{50.5-50}{5}\right)$=Pr(Z<0.1)=0.5398

*D11.a) Find E(X).

q=2/3 miss
p=1/3 get shot in
n=18

E(X)=np=18(1/3)=6
b) Find $\sigma(X)$.

$$\sigma = \sqrt{npq} = \sqrt{6\left(\frac{2}{3}\right)} = 2$$

c) Find the probability of getting between 8 and 10 shots (inclusive) in the net.

$\Pr(8 \leq X \leq 10) = \Pr(7.5 < Y < 10.5)$ Use the number line to find Y

$$Z = \frac{Y - \mu}{\sigma}$$

$= \Pr\left(\frac{7.5-6}{2} < Z < \frac{10.5-6}{2}\right)$=Pr(0.75<Z<2.25)
= Pr(Z<2.25) - Pr(Z<0.75)=0.9878 - 0.7734=0.2144

E. Methods of Sampling

Example 1. b) stratified because we are doing a random sample of each group or "strata"

Example 2. c) systematic

Example 3. a) cluster

Random Digits

Example 1. Choose numbers 6,9,0,4 and call Wendy's, Taco Bell, Archies and Jack Astors (order doesn't matter)

Example 2. As soon as there are more than 10 restaurants, we would need to use two digits to label them

01Wendy's
02Tim Hortons
03Swiss Chalet
04Burger King
05Jack Astors
06McDonald's
07Archie's
08East Side Marios
09Montanas
10Taco Bell
11Harveys
12Pizza Hut
13Red Lobster

If we use the same random list of numbers, we need to look for two-digit numbers that APPEAR in our list. A number such as the first two-digit number "69" doesn't apply because we don't have any restaurant labeled 69.
69043 81235 90721 30174 97245

Which restaurants would we select if we wanted a random sample of four restaurants?

Circle two-digit numbers until you get four numbers that are in the list from 01 to 13.

04, 12, 13, 01 are the first four numbers, so we would choose Burger King, Pizza Hut, Red Lobster and Wendy's.

Example 3.

Subjects must all have the same number of digits...they will be 01, 02, ..., 16

Circle two numbers at a time, without skipping any, until you find four people

81**05**7   271**02** 56**02**7   55892   33**06**3   41842   81868   7**10**35   43367

We got 02 twice, but you can't count the same person twice, so we get 05, 02, 06 and 10

Practice Exam Questions

E1. The answer is (c).

E2. The answer is (d).

E3. The answer is (c).

E4. The answer is (d).

E5. The answer is (d).

E6. The answer is (b).

E7. The answer is (e).

E8. The answer is (e).

E9. The answer is (b).

E10. The answer is (c).

E11. The answer is (c).

E12. The answer is (b).

E13. The answer is (c).

E14. The answer is (b).

E15. The answer is (d).

E16. The answer is (a).

E17. The answer is (d).

E18. The answer is (d).

E19. The answer is (a).

E20. The answer is (d).

E21. The answer is (d).

E22. The answer is (a).

E23. The answer is (b). The two factors or explanatory variables are temperature and humidity.

E24. The answer is (c). We have 3 temperatures and 2 humidities, so 2(3)=6.

E25. The answer is (c).

E26. The answer is (c).

E27. The answer is (a).

E28.
11793 20495 05907 11384 44982 20751 27498 12009
Circle one number at a time and do so until you obtain three names

1,1,7,9...we would use 1, 7, 9 since we can't pick 1 twice...so, call, Chapman, Stamm and Wright
The answer is c).

E29.
81507 27102 56027 55892 33063 41842 81868 71035 09001
The first four to get the new medication are
8,1,5,7 since 0 doesn't represent a name

Then, we get 2, 7, 1, 0, 2, 5, 6, 0, 2, 7, 5, 5, 8, 9, 2, 3, 3, 0, 6, 3, 4... The bolded ones are the ones we take since all others are repeats of the first four subjects who are already getting the medication
So, 2, 6, 3, 4 are the subjects to get the placebo...meaning, Chapman, Lovett, Dennis and Fitzgerald...note since there are only 8, we could just assume it was the four people we didn't get at the start, but since there could be 30 people, you need to know the method
The answer is (d).

E30.
A). The explanatory variable is the herbal tea. The answer is (b).
B). The confounding variable isn't a variable being studied, but any that will mess up your study and make the cause and effect difficult to prove. Since the elderly might be doing better from having extra visits and attention, their increased cheerfulness might be due to the company and have nothing to do with the tea.
The answer is (d).

E31.
14 42 92 60 56 31 42 48 03 71 65 10 36 22 53 22 49 06
We would pick two numbers at a time, from left to right, until we get 5 numbers that are between 01 and 30
14, 42, 31, 48, 03...since we don't count 42 twice

The answer is (c).

F. Sampling Distributions

Example 1.

$total\ \ 82\ kg$

$$\bar{x} = \frac{82}{20} = 4.1$$

$$Z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}}$$

$$\Pr(\bar{x} < 4.1) = \Pr\left(Z < \frac{4.1-4.2}{1.05/\sqrt{20}}\right)$$
$$= \Pr(Z < -0.43)$$
$$= 0.3336$$

Example 2.

$\mu = 10.35$ $\sigma = 0.8$ $n = 100$

$$Z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}}$$

$$\Pr(\bar{x} > 10.45) = \Pr\left(z > \frac{10.45-10.35}{\frac{0.8}{\sqrt{100}}}\right)$$
$$= \Pr(z > 1.25)$$
$$= 1 - 0.8944 = 0.1056$$

Example 3.

This adjustment is called the FINITE CORRECTION FACTOR

When $\frac{n}{N} > 0.05, the\ following\ equation\ should\ be\ used$:

** Look for questions where the size of the population N is given

Check $\frac{n}{N} = \frac{80}{800} > 0.05$

$$\sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{800-80}{800-1}} = 0.949276781$$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}}$$

$$\Pr(\bar{x} < 87) = \Pr\left(Z < \frac{87-84}{0.949276781(\frac{9}{\sqrt{80}})}\right)$$

$$= \Pr(Z < 3.16) = 0.9992$$

Example 4. The answer is D).

Example 5. The answer is B).

Practice Exam Questions on Sampling Distributions

F1. NOTE: F1 does not ask about a mean, average, or total, so it is just the original formula we talked about!!! $Z = \frac{X-\mu}{\sigma}$

(a) Let X be the diameter of a ping pong ball. Then $X \sim \mathrm{N}(\mu, \sigma^2)$ with $\mu = 33.0$, $\sigma = 1.0$

$$\Pr(32.5 < X < 33.0) = \Pr\left(\frac{32.5-33.0}{1.0} < Z < \frac{33.0-33.0}{1.0}\right) = \Pr(-0.5 < Z < 0.0)$$

$$= \Pr(Z < 0.0) - \Pr(Z < -0.5) = 0.5000 - 0.3085 = 0.1915.$$

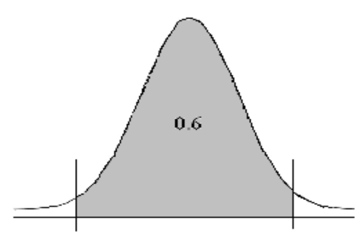
(b) $$\Pr(33.3 < X < 33.8) = \Pr\left(\frac{33.3-33.0}{1.0} < Z < \frac{33.8-33.0}{1.0}\right) = \Pr(0.3 < Z < 0.8)$$

$$= \Pr(Z < 0.8) - \Pr(Z < 0.3) = 0.7881 - 0.6179 = 0.1702.$$

(c) $\Pr(-z < Z < z) = 0.6 \Rightarrow 2\Pr(0 < Z < z) = 0.6$

1-0.60=0.40 total on both sides, which means 0.20 on each side

Total area below the line is then 0.80…look up 0.80 in the body



$\Rightarrow \Pr(Z < z) = 0.8.$

$\Rightarrow z = 0.84.$

The two z-scores are $z = \pm 0.84$, so since $x = \mu + z\sigma$, the two diameters are

$$\mu - 0.84\sigma = 33.0 - 0.84 \cdot (1.0) = 32.16$$

and $$\mu + 0.84\sigma = 33.0 + 0.84 \cdot (1.0) = 33.84.$$

That is $\Pr(32.16 < X < 33.84) = 0.6$, so 60% of the ping pong balls will have diameters between 32.16 mm and 33.84 mm.

F2. (a) Let X be the number of minutes using e-mail. Then $X \sim N(\mu, \sigma)$ with $\mu = 8$, $\sigma = 2$

The sample size is $n = 25$,

$$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{7.8-8}{2/\sqrt{25}} = -0.5$$

$$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{8.2-8}{2/\sqrt{25}} = 0.5$$

Therefore

$$\Pr(7.8 < \bar{X} < 8.2) = \Pr\left(\frac{7.8-8}{0.40} < Z < \frac{8.2-8}{0.40}\right) = \Pr(-0.5 < Z < 0.5)$$
$$= \Pr(Z < 0.5) - \Pr(Z < -0.5) = 0.6914 - 0.3085 = 0.3829.$$

(b) The sample size is now $n = 100$, $Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{7.8-8}{\frac{2}{\sqrt{100}}} = -1$

And $Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{8.2-8}{\frac{2}{\sqrt{100}}} = 1$

Therefore,

$$\Pr(7.8 < \bar{X} < 8.2) = \Pr\left(\frac{7.8-8}{0.20} < Z < \frac{8.2-8}{0.20}\right) = \Pr(-1.0 < Z < 1.0)$$
$$= \Pr(Z < 1.0) - \Pr(Z < -1.0) = 0.8413 - 0.1587 = 0.6826.$$

F3. The Central Limit Theorem says that the sampling distribution of sample means approaches to a normal distribution $N(\mu, \frac{\sigma}{\sqrt{n}})$ when the sample size gets large.

F4. (a)

Let X be the tuition of an undergraduate student. Then $\mu = \$4172$ and $\sigma = 525$.

$$\Pr(X < 4000) = \Pr\left(Z = \frac{X-\mu}{\sigma} < \frac{4000-4172}{525}\right) = \Pr(Z < -0.33) = 0.3707.$$

(b) $n = 36$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$\mathbf{Pr}(\bar{x} < 4000) = \Pr\left(Z < \frac{4000-4172}{\frac{525}{\sqrt{36}}}\right) = \Pr(Z < -1.97) = 0.0244$$

(c) The reason that the probability for part (b) is much lower than that in part (a) is because the sampling distribution of mean in part (b) has much smaller spread with a lot more values distributed near the centre than the population distribution in part (a). While few sample mean values, $\overline{X}$, are lower than 4000, there are many individual values, X, lower than 4000.

F5. $\mu = 1.5$ $\sigma = 0.5$ $n = 100$

$$Z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}}$$

$$\Pr(\bar{x} > 1.0) = \Pr\left(z > \frac{1.0-1.5}{\frac{0.5}{\sqrt{100}}}\right)$$
$$= \Pr(z > -10) = 1$$

(Note: the table doesn't go to -10, but the area below -3.49 is already almost 0, so the area above it is almost 1)

The answer is A)

F6. Assume that men's weighs are normally distributed…

$\mu = 172$ *and* $\sigma = 29, n = 25$

$\Pr(150 < \bar{x} < 180) = \Pr\left(\frac{150-172}{29/\sqrt{25}} < Z < \frac{180-172}{29/\sqrt{25}}\right) = \Pr(-3.79 < Z < 1.38)$

Use table 1 and look up the area…Pr(Z<1.38) – Pr(Z<-3.79)=0.9162-0.0002=0.916
So, the probability is 0.916

F7. (a)

Let X be the credit card balance. Then $X \sim \mathrm{N}(\mu, \sigma)$ with $\mu = 2780$, $\sigma = 900$. So

$$\Pr(X < 2500) = \Pr\left(Z = \frac{X-\mu}{\sigma} < \frac{2500-2780}{900}\right) = \Pr(Z < -0.31) = 0.3783.$$

(b) Now we are looking at the distribution for the sample mean $\overline{X}$ in a sample of size $n = 25$. Then $\overline{X} \sim \mathrm{N}(\mu_{\overline{X}}, \sigma_{\overline{X}})$ where the mean and standard deviation for $\overline{X}$ are given by $\mu_{\overline{X}} = \mu = 2780$ and $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{900}{\sqrt{25}} = 180$. Therefore,

$$\mathbf{Pr}(\bar{x} < 2500) = \Pr\left(Z < \frac{2500-2780}{\frac{900}{\sqrt{25}}}\right) = \Pr(Z < -1.56) = 0.0594$$

F8.

Now we are looking at the distribution for the sample mean $\overline{X}$ in a sample of size $n = 10$. Then $\overline{X} \sim \text{N}(\mu_{\overline{X}}, \sigma_{\overline{X}})$ where the mean and standard deviation for $\overline{X}$ are given by $\mu_{\overline{X}} = \mu = 625$ and $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{150}{\sqrt{10}} = 47.43$. Also, the total is given, so we must divide $7000 by 10 to get a mean of 700.

Therefore,

$$\mathbf{Pr}(\bar{x} < 700) = \Pr\left(Z > \frac{700-625}{\frac{150}{\sqrt{10}}}\right) = \Pr(Z > 1.58) = 1 - 0.9429 = 0.0571$$

F9. This adjustment is called the FINITE CORRECTION FACTOR

When $\frac{n}{N} > 0.05, the\ following\ equation\ should\ be\ used$:

** Look for questions where the size of the population N is given

N=350

$\mu = 150$

$\sigma = 35$

$n = 50$

$$\frac{\boldsymbol{n}}{\boldsymbol{N}} = \frac{\mathbf{50}}{\mathbf{350}} = \mathbf{0.142} > 0.05$$

$\frac{\sigma}{\sqrt{n}} = 35/\sqrt{50}$=4.949747468

$$\sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{350-50}{350-1}} = 0.92714554$$

$$Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}} = \frac{160-150}{4.949747468(0.92714554)} = 2.18$$

Pr(x̄<160)=(Z<2.18)=0.9854

G. Z-Based Confidence Intervals for a Population Mean

Example 1.
$n = 3$
$z_{\alpha/2} = 1.645\ (90\%\ of\ CI)$
$\bar{x} = 3.2$
$\sigma = 0.2$
$\bar{x} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) = 3.2 \pm 1.645\left(\frac{0.2}{\sqrt{3}}\right) = 3.2 \pm 0.1899 = (3.01, 3.39)$

Example 2. **(a)** Set up a 99% confidence interval for the true population mean amount of paint contained in 5-litre cans.

$\sigma = 0.1,\ n = 50,\ \bar{x} = 4.975.$

$\alpha = 0.01,\ z_{\alpha/2} = 2.576.$

The 99% confidence interval for μ is

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = \bar{x} \pm 2.576\frac{\sigma}{\sqrt{n}} = 4.975 \pm 2.576 \cdot \frac{0.1}{\sqrt{50}} = 4.975 \pm 0.0364$$

$= (4.939, 5.011).$

(b) No, the manager cannot complain to the manufacturer because he can be 99% certain that the true mean lies between $4.939\,\text{L}$ and $5.011\,\text{L}$.

Example 3.

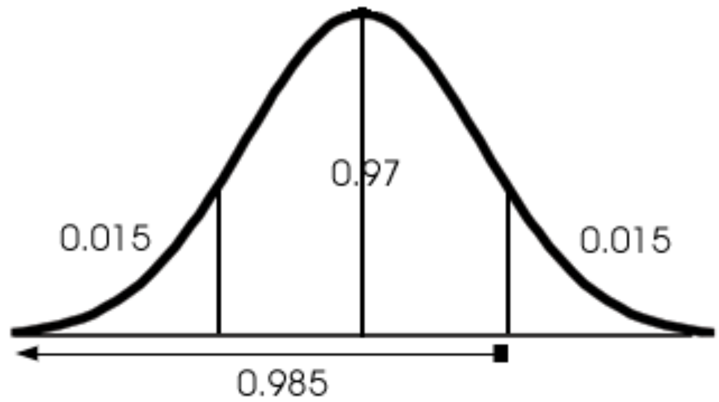
$$\bar{x} = \frac{18.52 + 21.48}{2} = 20$$

width $= 21.48 - 18.52 = 2.96$
$E = Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$
$1.48 = 1.96\left(\frac{\sigma}{\sqrt{50}}\right)$
$1.48 = 0.277186\sigma$
$\sigma = 5.34$

Example 4. $Z_{\alpha/2} = 1.645\ for\ a\ 90\%\ confidence\ interval$
New error$=$ $old\ E \div old\ Z^* \times new Z^*$$=1.48 \div Z^*\ 95\%\ CI \times Z^*$90%CI
$= 1.48 \div 1.96 \times 1.645 = 1.2$
$= (\bar{x} - E, \bar{x} + E)$
$= (20 - 1.24, 20 + 1.24)$
$= (18.76,\ 21.24)$

Example 5. I and III are true. So, the answer is B).

Example 6. 97% CI



Look up
0.985 $in\ body \quad Z_{crit} = \pm 2.17$

The answer is A).

Example 7. Check if 1.3 is in our 90% $(\alpha = 10\%)$ conf. interval
$(1.28,\ 1.34)$
It is $\therefore$ fail to reject H_o $at\ \alpha = 10\%$
(means p-value> $\alpha = 10\%$)
$\therefore p - value\ > 5\% \quad since\ it\ is\ > 10\%$
The answer is A).

Example 8.

$$E = Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) = 2.326\left(\frac{4}{\sqrt{100}}\right) = 0.9304$$

Practice Exam Questions on Z-Based Confidence Intervals for a Population Mean

G1. (a)

$$\bar{x} = \frac{190.5+189.0+195.5+187.0}{4} = \frac{762}{4} = 190.5.$$

$\sigma = 3.14$, $n = 4$.

$\alpha = 0.10$, $z_{\alpha/2} = 1.645$.

The 90% confidence interval for μ is

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = \bar{x} \pm 1.645\frac{\sigma}{\sqrt{n}} = 190.5 \pm 1.645 \cdot \frac{3.14}{\sqrt{4}} = 190.5 \pm 2.58$$

$$= (187.9, 193.1).$$

(b) The 90% confidence interval for μ would now be

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = \bar{x} \pm 1.645\frac{\sigma}{\sqrt{n}} = 190.5 \pm 1.645 \cdot \frac{3.14}{\sqrt{1}} = 190.5 \pm 5.17$$

$$= (185.3, 195.7).$$

(c) α has been changed from $\alpha = 0.10$ to $\alpha = 0.01$, so now $z_{\alpha/2} = 2.576$.

The 99% confidence interval for μ is therefore

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = \bar{x} \pm 2.576\frac{\sigma}{\sqrt{n}} = 190.5 \pm 2.576 \cdot \frac{3.14}{\sqrt{4}} = 190.5 \pm 4.04$$

$$= (186.5, 194.5).$$

G2. (a)

$\bar{x} = 540$
$\sigma = 80$
$n = 10$

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 540 \pm 1.96\frac{80}{\sqrt{10}}$$
$$= 540 \pm 49.58$$
$$= (490.42, 589.58)$$

(b)

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 540 \pm 2.575\frac{80}{\sqrt{10}}$$
$$= 540 \pm 65.14$$
$$= (474.86, 605.14)$$

This interval is wider since in order to be more confident that the interval contains the true population mean, we need a larger range at values.

G3. n=120, $\bar{x} = 18, \sigma = 3$ $\alpha = 0.05, so\ Z_{\alpha/2} = Z_{0.025} = 1.96$

The 95% confidence interval for μ is

$\bar{x} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$
$=18 \pm 1.96\frac{3}{\sqrt{120}}$
$=18 \pm 0.54$
=(17.46, 18.54)

G4. n=14, $\bar{x} = 65.12, \sigma = 24.60$ $\alpha = 0.05, so\ Z_{\alpha/2} = Z_{0.025} = 1.96$

The 95% confidence interval for μ is

$\bar{x} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$
$=65.12 \pm 1.96\frac{24.6}{\sqrt{14}}$
$=65.12 \pm 12.89$
=(52.23, 78.01)

G5. n=15, $\overline{x} = 29.50, \sigma = 12$ $\quad \alpha = 0.05, so\ Z_{\alpha/2} = Z_{0.025} = 1.96$

The 95% confidence interval for μ is

$$\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$=29.50 \pm 1.96 \frac{12}{\sqrt{15}}$

$=29.50 \pm 6.07$

=(23.43, 35.57)

G6. n=12, $\overline{x} = 24, \sigma = 3$ $\quad \alpha = 0.05, so\ Z_{\alpha/2} = Z_{0.05} = 1.645$

The 90% confidence interval for μ is

$$\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$=24 \pm 1.645 \frac{3}{\sqrt{12}}$

$=24 \pm 1.42$

=(22.6, 25.42)

G7.**(a)** $\sigma = 100$, $n = 64$, $\bar{x} = 350$.

$\alpha = 0.05$, $z_{\alpha/2} = 1.96$.

The 95% confidence interval for μ is

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = \bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}} = 350 \pm 1.96 \cdot \frac{100}{\sqrt{64}} = 350 \pm 24.5 = (325.5, 374.5)$$

(b) No, since the 95% confidence interval does not contain the claimed mean of 400 hours. We are 95% certain that the true mean lifetime is less than 400 hours.

G8.SE=$\frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{35}} = 0.68$

G9. (56, 68) is the confidence interval, so the width of the interval is
68 -56 = 12 and the margin of error is 1/2 width=1/2 (12) =6

G10. $\bar{x} = 36$ and $E = 1.3$

Use the formula
New E= old E $\div$ $old\ z_{\alpha/2} \times new\ z_{\alpha/2} = 1.3 \div 2.576 \times 1.645 = 0.83$

G11. n=25 and $\sigma = 100$
$\bar{x} = 450$
90% CI is $\bar{x} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

If we find a 95% confidence interval, the margin of error, $z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$ will be larger since the value of z^* will be greater...Also, a smaller sample size, means dividing by a smaller number, so that results in a larger margin of error, so E) is true

G12. a) look up the area 0.97 (0.94 total on both sides around 0 and so there is 0.0.3 on each side) on the Z-table...you get Z=1.88

b) look up the area 0.94 on the Z-table...you get Z=1.555

G13. $n = 20$, $\bar{x} = 1.67$, $\sigma = 0.32$.

$\alpha = 0.05$, $z_{\alpha/2}$=$z^* = 1.96$

The 95% confidence interval for μ is

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} = \bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}} = 1.67 \pm 1.96 \cdot \frac{0.32}{\sqrt{20}} = 1.67 \pm 0.14$$

=(1.53, 1.81)

G14. 95% CI (86.49, 89.49)

$$E = \frac{1}{2} width = \frac{1}{2}(89.49 - 86.49)$$
$$= \frac{1}{2}(3) = 1.5$$

$$\bar{x} = \frac{86.49 + 89.49}{2} = 87.99$$

$$\therefore 1.5 \div 1.96 \times 2.576 = 1.97 \quad newE$$

$$\therefore (87.99 - 1.971, 87.99 + 1.971) = (86.019, 89.961)$$

G15. 90% CI $E = 10$ $n = 600$

E= $z_{\alpha/2}(\frac{\sigma}{\sqrt{n}})$

$10 = 1.645(\frac{\sigma}{\sqrt{600}})$

$\sigma = 148.9$

$* G16. \mu = \bar{x} \pm Z_{\propto/2}\left(\frac{\sigma}{\sqrt{\text{n}}}\right)\sqrt{\frac{N-\text{n}}{N-1}}$

Here, he expects you to use the finite correction factor since

$\frac{\text{n}}{\text{N}} = \frac{63}{400} = 0.158 > 0.05$

$\therefore\ \mu = 211 \pm 1.96\left(\frac{48}{\sqrt{63}}\right)\sqrt{\frac{400-63}{400-1}}$

$\mu = 211 \pm 1.96\left(\frac{48}{\sqrt{63}}\right)\sqrt{\frac{337}{399}}$

$\mu = 211 \pm 10.89$

$\mu = (211 - 10.89, 211 + 10.89)$

$\mu = (200.11, 221.89)$

H. Finding the Sample Size and Margin of Error

Example 1.

$99\% \quad \therefore Z^* = 2.576$

$width = 6 \quad \therefore E = 3$

$\sigma^2 = 35 \quad \therefore \sigma = \sqrt{35} = 5.916$

$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{2.576(5.916)}{3}\right)^2 = 25.8$

$\therefore n = 26 \quad (round\ up)$

The answer is E).

Example 2.

$\sqrt{4} = 2\ times$

twice as wide, which means 4 times smaller in terms of sample size

The answer is A).

As n increases by 4 times, E decreases by $\sqrt{4} = 2\ times$

As n decreases by 4 times, E increases $\sqrt{4} = 2\ times$

Example 3. 90% $z_{\alpha/2} = 1.645 \quad width = 7$ E=width/2 = 7/2 = 3.5

min to max $35\ to\ 130 \quad \therefore range = 130 - 35 = 95$

$\therefore \sigma = \frac{range}{4} = \frac{95}{4} = 23.75$

$\left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = 124.6\ or\ 125\ cows$

Practice Exam Questions on Finding the Sample Size and Margin of Error

H1. $\sigma = 3.14, E = 2$ (within 2 pounds)

$\alpha = 0.05$, $z_{\alpha/2} = 1.96$.

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{1.96 \cdot 3.14}{2}\right)^2 = 9.47$$

Therefore a sample of size 10 would be required, i.e., he would need to weigh himself at least 10 times per month.

H2. **(a)** What sample size is needed? E= width/2= 5/2=2.5

n=$\left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{2.576(20)}{2.5}\right)^2 = 424.7$

∴ We'd need a sample size of 425.

(b) If 95% confidence is desired, what sample size is necessary?

n=$\left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{1.96(20)}{2.5}\right)^2 = 245.9$

∴ We'd need a sample size of 246.

H3.

n=$\left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{1.645(80)}{10}\right)^2 = 173.2$

∴ We'd need a sample size of 174.

H4. $\sigma = 15.8,\ E = 4$ (within 4 lbs)

$\alpha = 0.10, so\ z_{\alpha/2} = 1.645$

n=? n=$\left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{1.645(15.8))}{4}\right)^2 = 42.2$

∴ We'd need a sample size of 43.

H5. A) $\sigma = 54$, $E = 8$ (within 8 minutes)

$\alpha = 0.10, so\ Z_{\alpha/2} = 1.645$

n=? n=$\left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{1.645(54)}{8}\right)^2 = 123.3$

∴ We'd need a sample size of 124.

b) $\alpha = 0.01, so\ Z_{\alpha/2} = 2.576$

n=? n=$\left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{2.576(54)}{8}\right)^2 = 302.3$

∴ We'd need a sample size of 303.

I. Confidence Intervals for a Population Proportion (Ch. 8)

Example 1.

$95\%\ CI \quad \therefore Z_{\alpha/2} = 1.96 \quad n = 65$

$\hat{p} = \frac{28}{65} = 0.43 \qquad \hat{q} = 1 - 0.43 = 0.57$

$$E = Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$E = 1.96\sqrt{\frac{(0.43)(0.57)}{65}}$$

$$E = 0.12$$

width= $0.12 \times 2 = 0.24 \quad or\ 24\%$

Example 2.

$$SE = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{0.43(0.57)}{65}} = 0.06$$

Example 3.

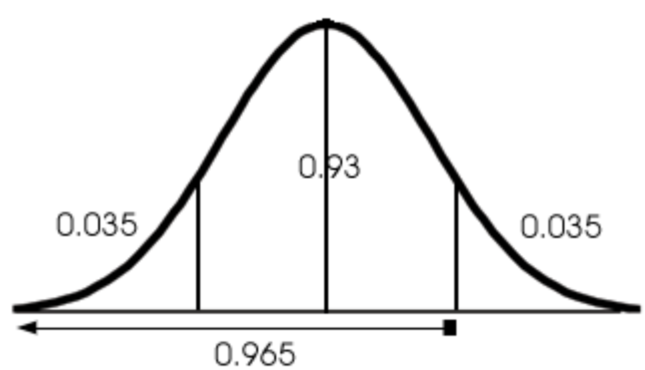
(0.34,0.60) within 12%, so E=0.12

Use p=0.6 (closest to 0.5)

Look up Area below 0.965 and get 1.81

$$n = \left(\frac{Z_{\alpha/2}}{E}\right)^2 p(1-p)$$

$$= \left(\frac{1.81}{0.12}\right)^2 (0.60)(0.40) = 54.6\ or\ 55$$



Example 4.

$p = 0.40 \qquad n = 1624 \qquad E = 0.02$

$$n = \left(\frac{Z_{\alpha/2}}{E}\right)^2 p(1-p)$$

$$1624 = \left(\frac{Z_{\alpha/2}}{0.02}\right)^2 (0.4)(0.6)$$

$$1624 = \left(Z_{\alpha/2}\right)^2 (600)$$

$$\left(Z_{\alpha/2}\right)^2 = 2.70\overline{6}$$

$$Z_{\alpha/2} = 1.645$$

$\therefore$ 90% confidence interval

The answer is A).

Example 5.

$E = 0.05$
$Z_{\alpha/2} = 2.576$ (99% CI)
$n = ?$

$p = 0.30$ ($closest\ to$ 0.5)

$$n = \left(\frac{Z_{\alpha/2}}{E}\right)^2 p(1-p)$$

$$n = \left(\frac{2.576}{0.05}\right)^2 (0.3)(0.7)$$

$$= 557.4 \qquad \therefore n = 558$$

The answer is B).

Practice Exam Questions Confidence Intervals for a Population Proportion

I1.
A) $\mu_{\hat{p}}$=p=0.50

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.5(1-0.5)}{200}} = 0.035$$

$$\sigma_{\hat{p}}{}^2 = 0.035^2 = 0.001225$$

b) $\mu_{\hat{p}}$=p=0.8

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.8(1-0.8)}{300}} = 0.02309$$

$$\sigma_{\hat{p}}{}^2 = 0.023^2 = 0.000533$$

I2. Check if $np \geq 5 \; and \; n(1-p) \geq 5$

np=100(0.4)=40 so yes, it is greater or equal to 5

n(1-p)=100(1-0.4)=60 so yes, it is too

Therefore, n is large enough

b) np=40(0.1)=4 which is NOT greater or equal to 5
Therefore, n is NOT large enough

I3. Recall, 95% of the data occurs within two standard deviations of the mean

n=30
p=0.5
$\mu_{\hat{p}}$=p=0.50

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.5(1-0.5)}{30}} = 0.0913$$

Therefore, 95% of all sample proportions lie within 2 standard deviations of the mean

Or use the confidence interval formula
($\mu_{\hat{p}}$-2σ, $\mu_{\hat{p}}$ +2σ)= (0.5-2(0.0913), 0.5 +2(0.0913) = (0.317, 0.683)

I4.
$\hat{p} = \frac{42}{70} = 0.6$

$Z_{\alpha/2}$=1.96 (95% CI)

n=70

$p = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
$=0.60 \pm 1.96\sqrt{\frac{0.6(0.4)}{70}}$
$=0.6 \pm 0.11$
=(0.49, 0.71)

I5.

$\hat{p} = \frac{25}{75} = 0.3333$

$Z_{\alpha/2}$=1.645 (90% CI)
n=75

$p = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

=$0.3333 \pm 1.645\sqrt{\frac{0.3333(0.66666)}{75}}$
=0.3333 ± 0.0895
=(0.24, 0.42)

I6.

$\hat{p} = \frac{102}{220} = 0.464$

$Z_{\alpha/2}$=1.96 (95% CI)
n=220
x=102

$p = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

=$0.464 \pm 1.96\sqrt{\frac{0.464(0.536)}{220}}$
=0.464 ± 0.0659
=(0.398, 0.53)

I7. a)

$$\hat{p} = \frac{145}{550} = 0.2636$$

$Z_{\alpha/2}$=1.96 (95% CI)
n=550
x=145

$$p = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$=0.2636 \pm 1.96\sqrt{\frac{0.2636(0.7364)}{550}}$
$=0.2636 \pm 0.0368$

b) $Z_{\alpha/2}$=2.576 (99% CI)

$$p = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$=0.2636 \pm 2.576\sqrt{\frac{0.2636(0.7364)}{550}}$
$=0.2636 \pm 0.0484$

c) 99% is wider than a 95% interval. We have to be more confident the true population proportino is in our interval.

I8.a)
E=0.06
$Z_{\alpha/2}$=1.645

$$n = \left(\frac{Z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{1.645}{0.06}\right)^2 (0.5)(0.5) = 187.9$$

So, the sample size needs to be 188.

b) E=0.06
$Z_{\alpha/2}$=1.96

$$n = \left(\frac{Z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{1.96}{0.06}\right)^2 (0.5)(0.5) = 266.8$$

So, the sample size needs to be 267.

c) E=0.04
$Z_{\alpha/2}$=1.96

$$n = \left(\frac{Z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{1.96}{0.04}\right)^2 (0.5)(0.5) = 600.25$$

So, the sample size needs to be 601.

I9.

$\hat{p} = \frac{85}{250} = 0.34$

$Z_{\alpha/2} = 1.96$

$p = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.34 \pm 1.96\sqrt{\frac{0.34(0.66)}{250}} = 0.34 \pm 0.0587 = (0.28, 0.399)$

$I10. \hat{p} = \frac{450}{1000} = 0.45$

$Z_{\alpha/2}$=1.96 (95% CI)

n=1000

$p = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

=$0.45 \pm 1.96\sqrt{\frac{0.45(0.55)}{1000}}$

=0.45 ± 0.0308

=(0.42, 0.48)

I11.

$p = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$\hat{p} = 0.75$

$standard\ error = 0.04 = \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$0.04 = \sqrt{\frac{0.75(0.25)}{n}}$

$0.04^2 = \frac{0.75(0.25)}{n}$

$0.0016n = 0.1875 \quad \therefore n = 117.2\ or\ 118$

The answer is B.

J. t-Based Confidence Intervals Mean: σ *unknown* (Ch. 8)

Example 1.

a) df=n-1=10-1=9 with 95% confidence...look it up on table 2

$t_{\alpha/2}$=2.262

b) n=20 df=20-1=19

$t_{\alpha/2}$=2.861

Example 2.

$\boldsymbol{\sigma\ uknown, use\ table\ 2}$

n=12, s^2=0.12,s=0.346 and $\bar{x} = 3\ with\ a$ 95% CI

df=12-1=11 and $t_{\alpha/2}$=2.201

$\mu = \bar{x} \pm t_{\alpha/2} \dfrac{s}{\sqrt{n}}$

$=3 \pm 2.201\left(\frac{0.346}{\sqrt{12}}\right)$

$=3 \pm 0.2198$

b) 99% CI , df=11, $t_{\alpha/2}$=3.106

$\mu = \bar{x} \pm t_{\alpha/2} \dfrac{s}{\sqrt{n}}$

$=3 \pm 3.106\left(\frac{0.346}{\sqrt{12}}\right)$

$=3 \pm 0.3102$

Example 3.

$n = 50 \quad \bar{x} = 29 \quad s = 5.3 \quad$ 99%

$df = 50 - 1 = 49\ use\ 40df$

$t_{\alpha/2} = 2.704$

(use df=40)

$\mu = \bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) = 29 \pm 2.704\left(\frac{5.3}{\sqrt{50}}\right) = 29 \pm 2.027 = (26.97, 31.03)$

Example 4. $\bar{x} = \frac{110.69+129.31}{2} = 120$

Width = $129.31 - 110.69 = 18.62$

$E = \frac{1}{2}(18.62) = 9.31$

$s^2 = 342.25 \quad \therefore s = 18.5$

$E = t_{\alpha/2}(\frac{s}{\sqrt{n}})$

$9.31 = t_{\alpha/2}(\frac{18.5}{\sqrt{30}})$

$t_{\alpha/2} = 2.756$

Go to 29 df and look for 2.756 and get 99%.

The answer is B).

Example 5. It can't be determined since it doesn't say normal distribution and $n = 10 \ (too\ small)$

If $n < 15$, must be approximately symmetrical and no outliers to use t test

Practice Exam Questions t-Based Confidence Intervals

J1. $\boldsymbol{\sigma\ uknown, use\ table\ 2}$

$n = 10$, $\bar{x} = 260$,s=140, df=10-1=9

$\alpha = 0.10$, $t_{n-1,\alpha/2} = t_{9,0.05} = t^* = 1.833$.

The 90% confidence interval for μ is

$\mu = \overline{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$

$=260 \pm 1.833\left(\frac{140}{\sqrt{10}}\right)$

$=260 \pm 81.15$

=(178.85, 341.15

J2. **(a) Nearly Normal condition:** We don't have the actual data, but since the sample of 45 weekdays is fairly large, it is okay to proceed.(n is greater or equal to 30)

(b) n=45**,** $\bar{x} = 125$, s=20. Since we want a 90% confidence interval, we know alpha=100-90=10% or 0.10

90% CI, df=n-1=45-1=44, t critical=1.684 .(use 40df)

The sampling distribution of the mean can be modeled by a Student's *t* model, with 44 degrees of freedom. (use 40df)

We will use a one-sample *t*-interval with 90% confidence for the mean daily income of the parking garage. The 90% confidence interval for μ is therefore

$\mu = \overline{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}}$

$=125 \pm 1.684\left(\frac{20}{\sqrt{45}}\right)$

$=125 \pm 5.02$

=(119.98,130.02)

(c) We are 90% confident that the interval $119.98 to $130.02 contains the true mean daily income of the parking garage.

(d) Since the interval contains $130 predicted by the consultant, so the consultant is likely correct

J3. n=15, s=18.70 and $\overline{x} = 87.25\ with\ a\ 95\%\ CI$

df=15-1=14 go along from 14df and look up (100-95)/2= 0.025 in one tail

We get $t_{\alpha/2}$=2.145

$\mu = \overline{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}}$

$=87.25 \pm 2.145\left(\frac{18.70}{\sqrt{15}}\right)$

$=87.25 \pm 10.36$

J4. n=19, s=18.50 and $\overline{x} = 65\ with\ a\ 90\%\ CI$

Go across from df=19-1=18 and look at (100-90)/2=0.05 in one tail

We get $t_{\alpha/2}$=1.734

$\mu = \overline{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}}$

$=65 \pm 1.734\left(\frac{18.50}{\sqrt{19}}\right)$

$=65 \pm 7.36$

J5. $\boldsymbol{\sigma}$ ***uknown, use table 2***

n=18, s=0.26 and $\bar{x} = 13.90$ *with a* 95% *CI*

df=18-1=17 and $t_{\alpha/2}$=2.131 (two -sided, so 0.025 in one tail or 0.05 in two tails)

$\mu = \bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$

$=13.90 \pm 2.110\left(\frac{0.26}{\sqrt{18}}\right)$

$=13.90 \pm 0.129 = (13.77, 14.03)$

We are 95% confident the true mean lies in this interval

K. **Chi Square Confidence Intervals (Ch. 8)**

Example 1.

$$n = 12 \quad df = n - 1 = 11 \quad \alpha = 10\% \quad \frac{\alpha}{2} = 0.05$$

$\frac{(n-1)s^2}{x^2} < \sigma^2 < \frac{(n-1)s^2}{x^2}$

$n-1, \frac{\alpha}{2}$ $\quad n-1, 1-\frac{\alpha}{2}$

$=11, 0.05$ $\quad = 11, \ 0.95$

$\frac{11(3.45)^2}{19.675} < \sigma^2 < \frac{11(3.45)^2}{4.575}$

$6.65 < \sigma^2 < 28.62$

$\therefore 2.58 < \sigma < 5.35$

is the CI for the pop. standard deviation.

Example 2.

$$\frac{(n-1)s^2}{{\chi^2}_{n-1,\frac{\alpha}{2}}} < \sigma^2 < \frac{(n-1)s^2}{{\chi^2}_{n-1,1-\frac{\alpha}{2}}}$$

$$\frac{(26)(2.62)^2}{{\chi^2}_{26,0.025}} < \sigma^2 < \frac{(26)(2.62)^2}{{\chi^2}_{26,0.975}}$$

$$\frac{(27)(2.26)^2}{41.923} < \sigma^2 < \frac{(27)(2.62)^2}{13.844}$$

$$4.26 < \sigma^2 < 12.89$$
$$2.06 < \sigma < 3.59$$

Practice Exam Questions on Chi Square

K1.
n=35
s=12.5
df=34 (use 30df)
90% CI

1- 0.90 = 0.10
$\frac{\alpha}{2} = \frac{0.10}{2} = 0.05$

$$\frac{(n-1)s^2}{{\chi^2}_{n-1,\frac{\alpha}{2}}} < \sigma^2 < \frac{(n-1)s^2}{{\chi^2}_{n-1,1-\frac{\alpha}{2}}}$$

$$\frac{(34)(12.5)^2}{{\chi^2}_{30,0.05}} < \sigma^2 < \frac{(34)(12.5)^2}{{\chi^2}_{30,0.95}}$$

$$\frac{(34)(12.5)^2}{43.773} < \sigma^2 < \frac{(34)(12.5)^2}{18.493}$$

$$121.36 < \sigma^2 < 287.27$$
$$11.02 < \sigma < 16.95$$

K2.n=30 s=5.8 df=29 95% CI

1- 0.95 = 0.05
$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$

$$\frac{(n-1)s^2}{{\chi^2}_{n-1,\frac{\alpha}{2}}} < \sigma^2 < \frac{(n-1)s^2}{{\chi^2}_{n-1,1-\frac{\alpha}{2}}}$$

$$\frac{(29)(5.8)^2}{{\chi^2}_{29,0.025}} < \sigma^2 < \frac{(29)(5.8)^2}{{\chi^2}_{29,0.975}}$$

$$\frac{(29)(5.8)^2}{45.722} < \sigma^2 < \frac{(29)(5.8)^2}{16.047}$$

$$21.34 < \sigma^2 < 60.79$$
$$4.62 < \sigma < 7.8$$

K3. n=10
$s^2=4.4$
df=9
90% CI

1- 0.90 = 0.10
$\frac{\alpha}{2} = \frac{0.10}{2} = 0.05$

$$\frac{(n-1)s^2}{\chi^2{}_{n-1,\frac{\alpha}{2}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2{}_{n-1,1-\frac{\alpha}{2}}}$$

$$\frac{(9)(4.4)}{\chi^2{}_{9,0.05}} < \sigma^2 < \frac{(9)(4.4)}{\chi^2{}_{9,0.95}}$$

$$\frac{(9)(4.4)}{16.919} < \sigma^2 < \frac{(9)(4.4)}{3.325}$$

$$2.34 < \sigma^2 < 11.91$$
$$1.53 < \sigma < 3.45$$

L. Hypothesis Testing (Ch. 9)

Example 1. $n = 32 \quad \sigma = 18.6 \quad \bar{x} = 108$

$H_o\ \mu = 100$

$H_a\ \mu \neq 100$ (2 *sided*)

$Z\ test = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{108-100}{\frac{18.6}{\sqrt{32}}} = 2.43$

$p\ value = 2(1 - 0.9925) = 0.015$

$At\ 1\ \%, p - value\ >\ \alpha\ \ fail\ to\ reject\ H_o$

$At\ \alpha = 5\%, p - value\ < \alpha\ \ reject\ H_o$

The answer is C).

Example 2. $n = 20 \quad \bar{x} = 580 \quad \sigma = 35$

$H_o\ \mu \geq 600 \qquad H_a\ \ \mu < 600$ (1 *sided*)

The answer is C).

b) $Z\ test = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{580-600}{\frac{35}{\sqrt{20}}} = -2.56$

$p - value = 0.0052$

The answer is D).

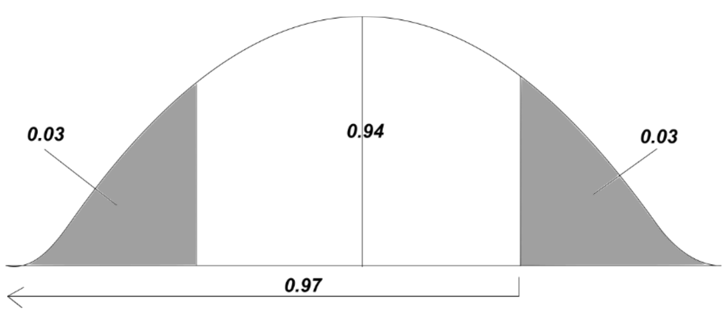
Example 3. $0.05 < p - value < 0.08$

$\therefore p - value > 5\% = \alpha$

$\therefore fail\ to\ reject\ \ H_o$

From your booklet p. 126, the answer is B).

Example 4. $\alpha = 6\% \div 2 = 0.03\ on\ each\ side$

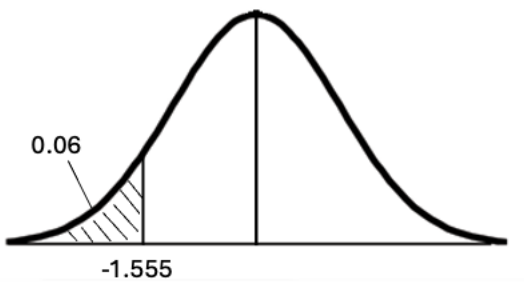


Look up Area 0.03 *in body of Z table*

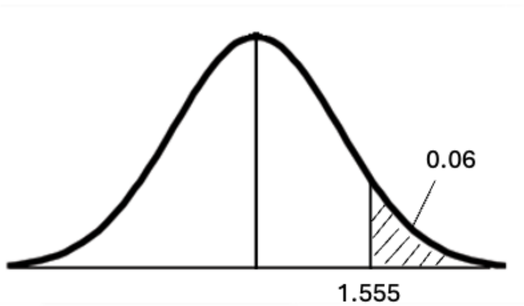
Rejection regions ± 1.88 and the answer is B).

What are your rejection regions for a 1-sided less than test and a 1-sided greater than test?

Less than test $\propto = 0.06$



Look up Area 0.06 in body of Z table
$Z = -1.555$
Greater than test



$Z = 1.555$

Example 5. I. fail to reject H_o since $\mu = 32$ is in our CI $(28.1,\ 35.7)$
II. reject H_o since H_o $\mu = 42$ is not in our CI.
∴ The answer is C).

Example 6.a)
H_o $\mu = 10$ H_a $\mu < 10$
$n = 26$ $\bar{x} = 5.5$ $p - value = 0.0025$
Look up area 0.0025 in the body and get Z= -2.81
$z\ test = -2.81$ $z\ test = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$
$\therefore -2.81 = \frac{5.5-10}{\sigma/\sqrt{26}}$
$-0.551086\sigma = -4.5$
$\sigma = 8.17$

Example 7. The answer is A).

We reject H0 if p-value is less than alpha

Example 8. $H_o \ \mu = 13 \qquad H_a \ \mu > 13$
$n = 16 \quad \bar{x} = 14.5 \quad \sigma = 2.2$

$z\ test = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

$z_{test} = \frac{14.5-13}{2.2/\sqrt{16}} = 2.73$

$p - value = 1 - 0.9968 = 0.0032$

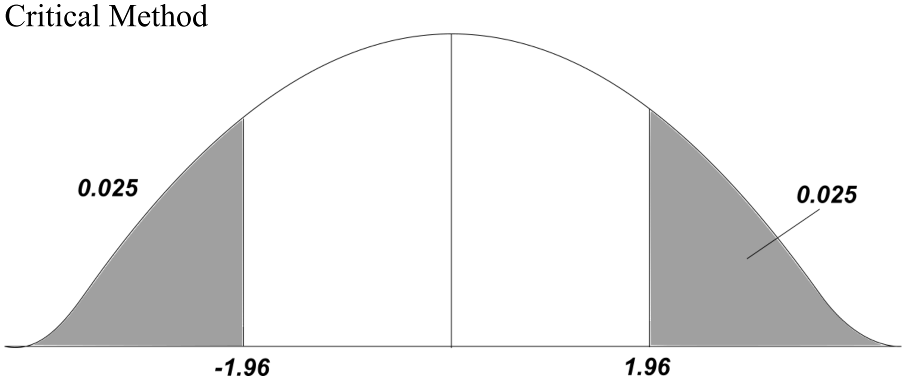
$at\ \alpha = 1\%, 2\%, 5\ or\ 10\% \quad p - value \ < \ \alpha \ reject\ H_o$

Example 9. $Z_{test} = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{22.05-21.25}{4.25/\sqrt{35}} = 1.11$

$H_o\ \mu = 21.25 \qquad H_a\ \mu \neq 21.25\ \ (2\ sided)$

$p - value = 2(\Pr(z < -1.11)$
$= 2(0.1335)$
$= 0.267$

Critical Method



$Z_{crit} = Z_{\frac{\alpha}{2}} = Z_{\frac{0.05}{2}} = Z_{0.025} = \pm 1.96$, and if you plot Z test = 1.11, it is outside of the two rejection regions.

The answer is A).

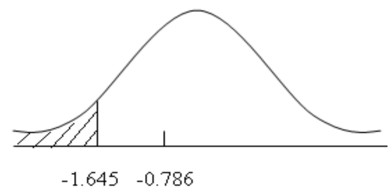
Practice Exam Questions on Hypothesis Testing

L1. **(a)** $H_0: \mu = 250$ vs. $H_a: \mu < 250$.

(b)

$n = 50$, $\bar{x} = 249.5$, $\sigma = 4.5$, $\mu_0 = 250$.

$\alpha = 0.05$, $z_{\alpha/2} = 1.645$.



$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{249.5 - 250}{4.5/\sqrt{50}} = \frac{-0.5}{0.64} = -0.79.$$

Since this is a one-tailed test and $z = -0.79$ is not smaller than the critical value $-z_\alpha = -1.645$ ($z > -z_{\alpha/2}$), therefore we fail to reject H_0. There is not enough evidence to suggest that the mean amount dispensed is less than 250 millilitres.

Alternatively, the *p*-value is $p = \Pr(Z < -0.79) = 0.2148$.

Since the *p*-value p=0.2148is not less than $\alpha = 0.05$, we do not reject the null hypothesis.

L2. H_0 $\mu = 500$
H_a $\mu > 500$

n=50, σ=90, $\bar{x} = 530$

$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{530-500}{90/\sqrt{50}} = 2.36$

Method 1: Draw the rejection region and see if Z=2.36 lies in it
Look up area 0.95 in the body (0.05 above the line on the right)

Reject H_0 if Z>1.645 at $\alpha = 0.05$

Since Z=2.36>1.645, we reject H_0

Therefore, there is reason to believe the process increases the yield.

Method 2: Calculate the p-value

Area=p-value= Pr(Z>2.36) = 1 - 0.9909 = 0.0091

p-value<0.05 and we reject H_0

L3. H_0 $\mu = 14$
H_a $\mu < 14$

n=16, σ=0.25, $\bar{x} = 13.75$

$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{13.75-14}{0.25/\sqrt{16}} = -4$

Method 1: See if Z= -4 lies in the rejection region
Draw the area 0.05 to the left and look up area 0.05 in the body

Reject H_0 if Z<-1.96 at $\alpha = 0.05$ and it is a one-sided interval

Since Z=-4< -1.645,So, Z test statistic is in the rejection region
So, we reject H_0

Method 2: Calculate the p-value for Z<-4 and if p-value < 0.05 we reject H_0
p-value=Area below -4= Pr(Z< -4)<0.0002
p-value< alpha=0.05 and we reject H_0
Therefore, there is evidence that the mean weight is less than 14 ounces.

L4.a)

(a) $H_o : \mu = 375$, $H_a : \mu \neq 375$ (2 sided test)

(b) $Z = \frac{350-375}{100/\sqrt{64}} = -2$

p-value=2(0.0228)=0.0456< alpha=0.05

So, we reject H_0...evidence the mean is different than 375

(c) Set up a 95% confidence interval for the true population mean lifetime of light bulbs in this shipment.

$\sigma = 100$, $n = 64$, $\bar{x} = 350$.

$\alpha = 0.05$, $z_{\alpha/2} = 1.96$.

The 95% confidence interval for μ is

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = \bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}} = 350 \pm 1.96 \cdot \frac{100}{\sqrt{64}} = 350 \pm 24.5 = (325.5, 374.5)$$

L5.
$H_0: \mu = 192$
$H_a: \mu > 192$ (1 sided)

$\alpha = 0.05$
$n = 100$
$\bar{x} = 196$
$\sigma = 16$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{196 - 192}{16/\sqrt{100}} = 2.5$$
$\Pr(Z > 2.5) = 1 - 0.9938 = 0.0062 < \alpha$

$\therefore reject\ H_0$

∴ statistically significant evidence the cholesterol is higher

L6.
$H_0: \mu = 7.8$
$H_a: \mu < 7.8$

$n = 50$
$\bar{x} = 7.4$
$\sigma = 0.6$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{7.4 - 7.8}{0.6/\sqrt{50}} = -4.71$$
$\Pr(Z < -4.71) = 0 < 1\%$

$\therefore reject\ H_0$

$\therefore yes$, there is stat. evidence that smokers need less sleep

L7.
$p - value = 0.02 \quad \alpha = 0.05$
$\therefore p - value < \alpha \quad \therefore reject\ H_0 \quad \therefore the\ result\ is\ significant$
The answer is C).

L8. a) State the null and alternative hypotheses.

$H_o : \mu = 32$

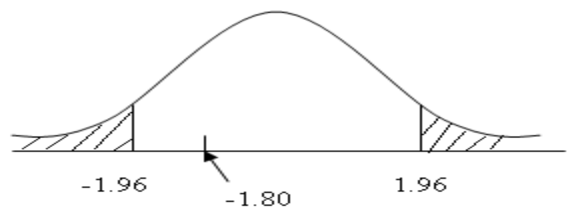
$H_a : \mu \neq 32$

b) Is there evidence that the machine is not meeting the manufacturer's specifications for average breaking strength? (Use a 5% level of significance.)

$\sigma = 2.72$

$$z = \frac{\bar{x} - \mu_o}{\sigma / \sqrt{n}} = \frac{31.3 - 32}{2.72 / \sqrt{50}} = -1.82$$

*picture should show -1.82



∴Fail to reject H_o.

∴ There is not sufficient evidence to support the claim that the machine is not meeting the manufacturer's specifications.

c) Compute the p-value and interpret its meaning.

$$p - value = 2p(z < -1.80) = 2(0.0344) = 0.0688$$

0.0688>alpha=0.05

So, fail to reject H0, so no evidence the mean is different than 32kg.

L9.

$\bar{x} = \frac{1015}{9} = 112.8 \quad , n = 9$

$H_0 : \mu = 100$

$H_a : \mu > 100 \quad (1 - sided) \qquad \sigma = 16$

$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{112.8 - 100}{\frac{16}{\sqrt{9}}} = 2.4$

Pr(Z>2.4) = 1-0.9918=0.0082

$p - value = 1 - 0.9918 = 0.0082 < 1\%$

$\therefore p. 126\ in\ prep\ booklet\ ...\ reject\ H_0 \quad \therefore there\ is\ evidence\ of\ principals\ claim$(very strong evidence against H0)

L10.

$\sigma = 35 \quad H_0 \ \ \mu = 65$

$H_a \ \ \mu > 65 \ \ (1 - sided) \quad n = 36 \quad \bar{x} = 75$

$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{75-65}{\frac{35}{\sqrt{36}}} = 1.71$

$\Pr(z > 1.71) = 1 - 0.9564 = 0.0436$

$p. 126\ in\ prep\ booklet$

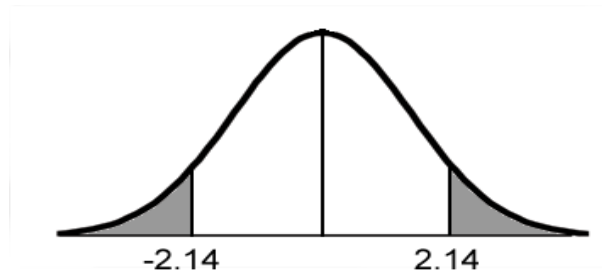
$\therefore p - value = 0.0436 < 5\% \quad \therefore reject\ H_0$

$\therefore yes, booking\ times\ are\ higher$ and there is strong evidence against H0.

$reject\ H_o\ if\ z\ test\ > 1.645$ (area above your line is 0.05 and area below is 0.95)
So, 1.71>1.645 so it is in a rejection region and so we reject H0.

L11. $H_o\ \ \mu = 20 \quad H_a\ \ \mu \neq 20$

$n = 238 \quad z = 2.14$



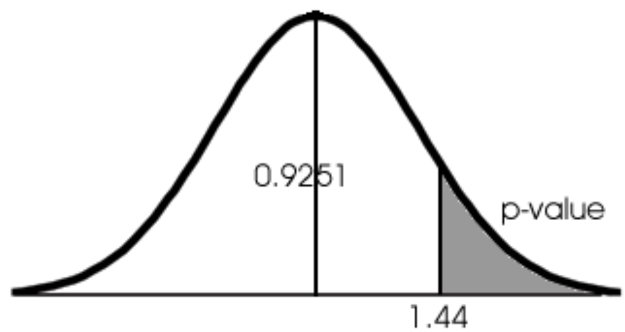
$p - value = 2\Pr(z < -2.14)$

$= 2(0.0162) = 0.0324$

The answer is B).

M. Z-tests for a Population Proportion

Example 1.



a) H_o $p = 0.60$
H_a $p > 0.60$ $(1 - sided)$
$\hat{p} = \frac{x}{n} = \frac{130}{200} = 0.65$

$Z\ test = \frac{\hat{p}-p_o}{\sqrt{\frac{p_o q_o}{n}}} = \frac{0.65-0.60}{\sqrt{\frac{0.60(0.40)}{200}}} = 1.44$

The answer is A).

b) p-value$= 1 - 0.9251 = 0.0749 > \alpha$
$\therefore$
$fail\ to\ reject\ H_o$ $(no\ statistical\ significance)$
The answer is A.

Example 2.

$\Pr(\hat{p} > k) = 0.20$

Look up 0.80 in body of z table $z = 0.84$

$H_0: p_0 = 0.75$
$n = 150$

$$z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

$$z = \frac{\hat{p}-0.75}{\sqrt{\frac{0.75(0.25)}{150}}}$$

$$\therefore 0.84 = \frac{\hat{p}-0.75}{0.0353553391}$$

$$0.0296984848 = \hat{p} - 0.75$$

$$\hat{p} = 0.78$$

$\therefore k = 78\%$

Practice Exam Questions on Z tests for a Population Proportion

M1.
H_0 $p_0= 0.85$
H_a $p_0\neq 0.85$ (2 sided p-value)

$$z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}=\frac{0.78-0.85}{\sqrt{\frac{0.85(0.15)}{100}}} = -1.96$$

p-value=2(0.025)=0.05
p-value$\leq$ 0.05...
∴reject H0 and there is statistical evidence the proportion is different than 85%.

M2.
$$\hat{p} = \frac{1150}{1500} = 0.7666$$

n=1500

$\alpha = 0.05$
$H_0: p \leq 0.75$
$H_a: p > 0.75$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$
$$Z = \frac{0.7666 - 0.75}{\sqrt{\frac{0.75(0.25)}{1500}}} = 1.49$$

P-value = Pr(Z>1.49) = 1- 0.9319=0.0681>0.05 so we fail to reject H0 and conclude there is no statistically significant evidence the percentage is greater than 75%.

M3. $H_0: p = 0.5$
$H_a: p > 0.5$ (winning means more than 50% of the votes)
$$\hat{p} = \frac{525}{1000} = 0.525$$

$\hat{q} = 0.475$
$\alpha = 0.01$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$Z = \frac{0.525 - 0.50}{\sqrt{\frac{0.5(0.5)}{1000}}} = 1.58$$

$\Pr(Z > 1.58) = 1 - 0.9429 = 0.0571 > \alpha$

$\therefore$ *fail to* reject H_0
$\therefore$ no statistical evidence the proportion is above 50%

M4.
$H_0 \quad p = 0.11$
$H_a \quad p > 0.11 \quad (1 \; sided)$

$$\hat{p} = \frac{x}{n} = \frac{175}{1500} = 0.1166666$$

$$z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.1166666-0.11}{\sqrt{\frac{0.11(0.89)}{1500}}} = 0.83$$

Pr(Z<0.83)=0.7967
$p - value = 1 - 0.7967 = 0.2033$> alpha=5%, so we would fail to reject H0 and conclude there is no statistical evidence of greater than 11%.

M5.
H_0 p_0=0.30
H_a p_0≠0.30

b) n=75

$$\hat{p} = \frac{x}{n} = \frac{25}{75} = 0.3333$$

$$z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.3333-0.30}{\sqrt{\frac{0.3(0.7)}{75}}} = 0.63$$

Pr(Z<-0.63)=0.2643
c) p-value=2(0.2643)=0.5286

d) p-value is huge! $\therefore$fail to reject H_0 and we can not conclude that the percent in favour is different than 30%.

M6.a) one sample t test
H_0 $\mu = 40$
H_a $\mu \neq 40$(2 sided)

$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{43-40}{8.5/\sqrt{65}}$=2.85
n=65

df=64

t crit= go down to 64 (use 60 df and across to find t=2.85)

2.660<2.85<3.232 and go up top and get the p-values

0.001(2)<p-value<2(0.005) remember to double them since it is a 2 sided test

0.002<p-value<0.01

Now, alpha=0.05 and the p-value<0.05, so we reject H_0 and concluded that the mean is significantly different than 40

b) H_0 p_0=0.20
H_a p_0<0.20 (1 sided)

b) n=65

$$\hat{p} = \frac{x}{n} = \frac{12}{65} = 0.1846$$

$$z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.1846-0.20}{\sqrt{\frac{0.2(0.8)}{65}}} = -0.31$$

Pr(Z<-0.31)=0.3783
p-value=0.3783>alpha=0.05

So, we fail to reject H_0 and conclude there is no stat. sign evidence that fewer than 20% purchase premium grade gasoline

M7. $p = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$\hat{p} = 0.75$

$standard\ error = 0.04 = \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$$0.04 = \sqrt{\frac{0.75(0.25)}{n}}$$

$$0.04^2 = \frac{0.75(0.25)}{n}$$
$$0.0016n = 0.1875 \quad \therefore n = 117.2 \; or \; 118$$

The answer is B.

M8. $H_0 \quad p = 0.20$

$H_a \quad p > 0.20 \quad (1 \; sided)$

$$\hat{p} = \frac{50}{200} = 0.25$$

$$z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.25-0.2}{\sqrt{\frac{0.2(0.8)}{200}}} = \frac{0.05}{0.028284} = 1.77$$

$p - value = 1 - 0.9616 = 0.0384$

The answer is D).

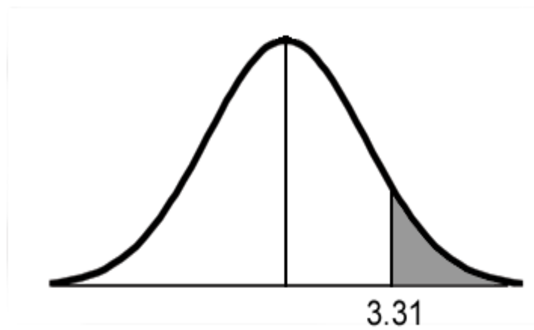
M9. $H_o \quad p = 0.65 \qquad H_a \quad p > 0.65$

$n = 1000 \quad \hat{p} = 0.70$

$$z_{test} = \frac{\hat{p}-po}{\sqrt{\frac{poqo}{n}}}$$
$$= \frac{0.70-0.65}{\sqrt{\frac{0.65(0.35)}{1000}}}$$
$$= 3.31$$

$\Pr(z > 3.31) = 1 - 0.9995 = 0.0005$

The answer is B).



M10. $H_o \quad p = 0.50 \qquad H_a \quad p > 0.50$

$\alpha = 0.05 \quad do \; not \; reject \; H_o$

$\therefore p - value > 0.05$

$\therefore p - value \; definitely > 0.02$

The answer is B).

N. t-Based Hypothesis Testing for Population Mean: σ *unknown* (Ch. 9)

Example 1. a) $H_o\ \mu = 14\,900$

$H_a\ \mu > 14\,900\ (1 - sided)$

b) $t\ test = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{16\,350-14\,900}{\frac{3999}{\sqrt{15}}} = 1.40$

c) $df = 14$ $\quad 1.345 < t\ test < 1.761$

$0.05 < p\ value < 0.10$

At 10% $\quad p\ value < \alpha = 10\%\quad reject\ H_o$

At 5% (or 1%) $\quad p\ value > \alpha\ \ fail\ to\ reject\ H_o$

d) The answer is B).

Example 2. a) $\frac{0.01}{2} = 0.005$

$n = 15 \quad \bar{x} = 49 \quad s = 12$

$df = n - 1 = 14$

Critical values= ± 2.977

(look up df=14 with 0.005 at top in each tail)

b) $t\ test = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{49-40}{\frac{12}{\sqrt{15}}} = 2.90$

go across 14 df $\quad 2.624 < \underset{2.90}{t\ test} < 2.977$

$0.005(2) < 2sided\ p < 0.01(2)$

$0.01 < p < 0.02 > \alpha = 1\%$

$\therefore fail\ to\ reject\ H_o$

The answer is A).

c) $H_o\ \mu = 40$

$H_a\ \mu > 40$

$0.005 < p < 0.01$

P value$< \alpha = 1\%\quad \therefore reject\ H_o$

The answer is D).

Example 3.

It can't be determined since it doesn't say normal distribution and

$n = 10\ (too\ small)$

If $n < 15$, must be approximately symmetrical and no outliers to use t test

Practice Exam Questions t-Based Hypothesis Tests

N1. a) Using a 1% level of significance, is there evidence that the population average is above $300?

Solution: since our alternative hypothesis is greater than, it is a one-sided p-value

$H_o \quad \mu = 300$ $\quad\alpha = 0.01$. df=99 (use 90)
$H_a \quad \mu > 300$

$$t_{test} = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{315-300}{\frac{45}{\sqrt{100}}} = 3.33 > 3.183$$

p-value <0.001< alpha=0.01 so we reject H0

Critical Method

df=n-1=99...one sided test, so look at 0.01 (one-sided) along the top of the t-table and 90 along the left and get critical t-value of 2.368. If your table doesn't have 99df, round down to the closest.... So, our rejection region is everything greater than 2.368. So, our t-statistic is 3.33 which is greater than 2.368, so we reject our null hypothesis.

b) What is your answer in (a) if the standard deviation is $75 and a 5% level of significance is used?

$$t_{test} = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{315-300}{\frac{75}{\sqrt{100}}} = 2 > 1.662$$

at 90df with 5% level of significance, we go down to 90df and across the top to t0.05 since it is a one-sided test and get t critical =1.662, so we would reject H0 if t test > 1.662. 2.0 lies inside the rejection region...therefore we reject H_0 and there is sufficient evidence that the population average is greater than $300.

p-value at 90df
1.987< t test = 2<2.368
0.01 <p value <0.025
p-value < alpha = 0.05 reject H0

N2.
$H_o \quad \mu = 220 \qquad\qquad \alpha = 0.05$
$H_a \quad \mu > 220$
$n = 22 \quad \bar{x} = 235 \qquad s = 25$
$df = 21$ look up on table

Critical method
$DR\ reject\ H_o\ \ if\ \ t_{test} > 1.721$ (21 df and alpha=0.05 in one tail)
$t_{test} = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{235-220}{\frac{25}{\sqrt{22}}} = 2.81 > 1.721$
$\therefore reject\ \ H_o$
$\therefore yes, prices\ are\ now\ higher$

P-value method
df=21
2.518< t test=2.81<2.831

0.005< one sided p value<0.01 < alpha=0.05
Reject H0, so there is evidence the cost increased

N3**.**

(a) Write appropriate hypotheses.
$H_o \quad \mu \geq 25 \qquad\qquad \alpha = 0.05$
$H_a \quad \mu < 25$
$n = 52 \quad \bar{x} = 24.25 \qquad s = 4.7 \quad df = 51$ look up on table (use 50)
(one-sided hypothesis)

(b) Compute the test statistic and the *p*-value.

$t_{test} = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{24.25-25}{\frac{4.7}{\sqrt{52}}} = -1.15$
$\therefore reject\ \ H_o$
Go across from 50 df and find t=1.15 and go up to the top to find the one-sided p-values

t =1.15<1.299

So, the p-value is p>0.10 and p-value > alpha = 0.05 so we fail to reject H0

(c) If the mean mileage of cars in the fleet is 25 mpg, then the chance that a sample mean of samples of size 52 is 24.25 mpg or less simply due to sampling error is greater than 10%.

(d) Since the *p*-value is greater than 5%, we fail to reject the null hypothesis. There is insignificant evidence that the mean mileage of cars in the fleet is less than 25 mpg.

N4.

H_0 $\mu = 50400$
H_a $\mu \neq 50400$
(valid or not, so two sided)

$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{50300-50400}{1000/\sqrt{400}} = -2$
n=400

df=399 (use 200df)

t crit= go down to 200df and across to find t=2

1.972<2<2.345 and go up top and get the p-values

0.01(2)<p-value<2(0.025) remember to double them since it is a 2 sided test

0.02<p-value<0.05

Now, p-value<10%, so we reject H0 at a 10% level of significance. (same at the 5% level)
p-value >0.01, so we fail to reject H0 at a 1% level of significance.

N5. H_0: $\mu = 15.5$
H_a: $\mu \neq 15.5$ (two-sided p-value)

$\mu = 15.5, n = 36, s = 2.55, \bar{x} = 14.75 \ and \ \alpha = 0.05$

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{14.75 - 15.5}{2.55/\sqrt{36}} = -1.76$$

df=36-1=35...go across from 35 df (use 30df) and find t=1.76

1.697<t=1.76<2.042
0.025(2)<p-value<0.05(2)
∴ 0.05<p-value<0.10

∴p-value> $\alpha = 0.05$
So, we fail to reject H_0, so the mean is not different than 15.5 kg.

b) 99% $t_{\alpha/2} = 2.75$ $df = 35$ $(use\ 30)$

$\mu = \bar{x} \pm t_{\alpha/2}(\frac{s}{\sqrt{n}})$

$= 14.75 \pm 2.75(\frac{2.55}{\sqrt{36}})$

$= 14.75 \pm 1.169 = (13.58, 15.92)$

N6. H_0: $\mu \leq 2$
H_a: $\mu > 2$ (one-sided p-value)

$\mu = 2, n = 45, s = 0.4, \bar{x} = 2.2$ and $\alpha = 0.05$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{2.2 - 2}{0.4/\sqrt{45}} = 3.35$$

df=45-1=44...go across from 44 df (use 40df) and find t=3.35

t=3.35>3.307
∴ p-value <0.001
∴p-value< $\alpha = 0.05$
So, we reject H_0 and therefore we reject the claim on the food label.

N7. a) $\bar{x} = 57 \quad s = 15 \quad n = 50$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$
$$= \frac{57 - 50}{\frac{15}{\sqrt{50}}}$$
$$= 3.2998$$

$b) H_o \quad \mu = 50 \qquad H_a \quad \mu \neq 50\ (2\ sided)$
$at\ df = 49 \quad (use\ 40)$
$2.704 < t = 3.2998 < 3.307$
$0.001(2) < 2\ sided\ p - value < 2(0.005)$
$0.002 < p - value < 0.01$
$\therefore p - value < 1\% \quad reject\ H_o$
$\therefore yes, evidence\ of\ a\ difference$

N8. We want to determine if the mean number of concurrent users is greater than 36.

95% confidence interval, so 100-95%=5% is the alpha....0.05

H_0 $\mu = 36$
H_a $\mu > 36$ (one-side p-value)

$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{37.5-36}{9.5/\sqrt{120}} = 1.73$

n=120, df=119...one sided, so look up df 100 along left and 0.05 along top of t-table and find critical value of t=1.66

So, the rejection region is everything in the area greater than 1.66. So, since 1.73 is above 1.66, we reject the null hypothesis.
Or use p-value 1.66<t=1.73<1.984
0.025<p value<0.05
P-value less than alpha=5% so we reject H0
t=1.73>1.66, so we reject the null hypothesis and therefore, there is significant evidence at the 5% level of significance that the mean number of users is greater than 36

N9.
H_0 $\mu = 100$
H_a $\mu \neq 100$

n=18

$\alpha = 0.02$

look up df=17 and 2-sided p-value = 0.02 and you get t=± 2.567.
The answer is B).

N10.
$H_0 \;\; \mu = 25 \qquad H_a \;\; \mu < 25 \;\; (reduce \;\therefore 1\; sided)$
$n = 81 \quad \bar{x} = 19 \quad s = 20 \quad \alpha = 0.01$
$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{19-25}{\frac{20}{\sqrt{81}}} = -2.7$
$df = 80 \;\; go\ across\ 80\ df,$
$2.639 < t = 2.7 < 3.195$
$0.001 < \; one - sided\ p - value < 0.005$
$\therefore p - value < \alpha \quad \therefore \; reject\ H_0$
$\therefore there\ is\ statistically\ significant\ evidence\ of\ reduced\ test\ anxiety$

or use critical method...alpha=0.01 and look up df=80 and get
t critical=- 2.374 since it is a less than test...reject H0 if t test <-2.374 and since
t test=-2.7< -2.374, we reject H0

N11.

$n = 20 \quad H_0 \;\; \mu = 5 \qquad H_a \;\; \mu \neq 5 \quad t = 1.45$

$df = 19 \rightarrow find\ t = 1.45$

$1.328 < 1.45 < 1.729 \;\; go\ up\ and\ find\ 2 - sided$

$p - values$

$0.05(2) < p - value < 0.10(2)$

0.10<p<0.20

$\therefore at\ \alpha = 0.10, p - value > \alpha \;\; \therefore fail\ to\ reject\ H_0$

$at\ \alpha = 0.05, p - value > \alpha \;\; \therefore fail\ to\ reject\ H_0$

The answer is D).

O. Type I and Type II Errors

Example 1. Power = 1 - Beta
0.80= 1- Beta
Beta= Pr(type II error) = 0.20

Example 2. Alpha and Beta (probability of type 1 and 2 errors) are inversely related
As $\beta \downarrow, \alpha \uparrow$ and as $\beta \downarrow, Power \uparrow$ since $Power = 1 - \beta$

I. true (as $\alpha \uparrow, \beta \downarrow so\ Power \uparrow$)
II. false (see p.172)
III. true
The answer is D).

Example 3.
A no
B no, alpha is fixed
C no, alpha is fixed
D yes, the p-value will be smaller than it should be as it should have been doubled

Example 4.
H_0 $\mu = 800$ vs. H_a μ <800

The mean weekly wage is actually $850 so that is NOT less than $800, so we know that H0 is true and there is no evidence Ha is true

Pr(not reject H0/H0 is actually true)
=1- Pr(Type I error)
=1-0.10
=0.90
The answer is D).

Practice Exam Questions on Type I and Type II Errors

O1. Type I= false positive= say the patient is unhealthy and really they are healthy

Type II= false negative= say patient is healthy and then they are born with an anomaly

O2.
Type I= reject H_0 and really H_0 is true...you say they are driving under the influence and really they aren't
Type II= fail to reject H_0 and really it is false...you assume the individual is not driving under the influence, but really they are

O3. Type I error= reject H_0 and really H_0 is true...say there is a difference in the average height, but really there isn't

Type II error= fail to reject H_0 and really it is false...conclude there is no difference in height, when really there is a difference

O4. When you fail to reject the null hypothesis based on your calculations, when really it is false, then you are making a type II error.

O5. When you reject the null hypothesis based on your data, but it is really true, you are making a type I error.

O6. A type 1 error is to reject H_0 when it is really true. So, in this case we would say the person has cancer, but really they don't. The answer is B).

A type 2 error is when you fail to reject the null hypothesis and it should have been rejected because it is false. So, in this case we would say the person doesn't have cancer, but they do in fact have it. So, the answer is A).

O7.
$H_o \quad \mu = 7 \quad H_a \quad \mu > 7$
$reject \ H_o \ \ H_o \quad true$
The answer is A).

O8. $H_o \quad \mu = 40 \quad H_a \ \mu > 40$
$reject \ H_o \quad H_o \ \ false$
The answer is C).

O9. $fail \ to \ reject \ H_o \quad H_o \ \ false$
$conclude = 9.4 \ hour \ but \ really \ it \ is \ > 9.4 \ hours$
The answer is C).

O10.

$\Pr\ (reject\ H_0\ /\ H_0\ true) = \Pr\ (type\ I\ error)$

$= 0.03$ ∴The answer is C).

O11.

$\Pr\ (not\ reject\ H_0\ /\ \ H_0\ true)$

$= 1 - \Pr\ (type\ I\ error)$

$= 1 - 0.02$

$= 0.98$ ∴ The answer is D).

$= 0.98$ ∴ The answer is D).

*O12.

$\Pr(type\ I\ error) = \alpha = same$

$\Pr(type\ II\ error) \downarrow as\ n\ \uparrow (power\ \uparrow)$

The answer is B).

*O13. I) as level of confidence $\downarrow, (1-\alpha) \downarrow and\ \alpha\ \uparrow,\ \therefore \beta, probability\ of\ type\ II\ error\ \downarrow$.

TRUE $(\alpha\ and\ \beta\ are\ inversely\ related)$

II)as $n\ \uparrow, E\ \downarrow$ ∴ the interval becomes narrower

FALSE

The answer is A).

P. Single Sample Test of a Variance (Ch. 9)

Example 1.

a) $H_o \;\; \sigma^2 = 4$

$H_a \;\; \sigma^2 < 4 \;\; (less\ than\ test)$

b) $n = 27 \quad s = 2.62 \quad \alpha = 5\%$

$x^2 test = \frac{(n-1)s^2}{\sigma^2} = \frac{26(2.62)^2}{4} = 44.62$

$x^2 crit = x^2 n - 1{,}1 - \alpha$

$= x^2 26, 0.95$

$= 15.379$

Reject H_o $if\ x^2 test < 15.379$

$x^2 test = 44.62 > 15.379$

$\therefore fail\ to\ reject\ H_o$

Using p-value:

Go across 26 df

41.923< 44.62 < 45.642

1-0.025 < p-value < 1- 0.01

0.975 < p-value < 0.99 and p-value >0.05 so we fail to reject H_0

Practice Exam Questions on Chi Square

P1.
a)
$H_0\ \sigma^2 = 45$

$H_a\ \sigma^2 < 45$ (one- sided so do NOT divide alpha by 2)

n=8
s^2=42
alpha=0.05 level of significance

χ^2 crit = $\chi^2{}_{n-1,1-\alpha}$=$\chi^2{}_{7,0.95} = 2.16735$...lower bound

DR.reject H_0 if $\chi^2\ test < 2.16735$
$\chi^2_{test} = \frac{(n-1)s^2}{\sigma^2} = \frac{7(42)}{45} = 6.53$

So, $\chi^2 test$ >2.16735 so we fail to reject H_0 and conclude there is NO sign. evidence the variance is less than 45

b)
$H_0\ \sigma^2 = 9$

$H_a\ \sigma^2 \neq 9$ (2 sided...divide alpha by 2)

alpha=0.10
n=20
s^2=18

χ^2 crit = $\chi^2{}_{n-1,\frac{\alpha}{2}} = \chi^2{}_{19,0.05} = 30.1435$..upper bound

χ^2 crit = $\chi^2{}_{n-1,1-\frac{\alpha}{2}}$=$\chi^2{}_{19,0.95} = 10.117$...lower bound

DR. Reject H_0 if $\chi^2\ test\ is > 30.1435\ or < 10.117$

$\chi^2_{test} = \frac{(n-1)s^2}{\sigma^2} = \frac{19(18)}{9} = 38$

So, $\chi^2\ test\ is\ greater\ than\ 30.1435\ and\ is\ in\ the\ rejection\ region, so\ we\ reject\ H_0$

So, there is stat. evidence the variability is different than the 9.

P2.
χ^2 crit = $\chi^2{}_{n-1,\alpha}$=$\chi^2{}_{19,0.05}$

$H_0\ \sigma^2 = 16$

$H_a\ \sigma^2 > 16$ (1- sided)

Using p-value

Go across from 19 df and find $\chi^2_{test} = \frac{(19)(23)}{16} = \frac{(n-1)s^2}{\sigma^2}$=27.31

27.204 <$\chi^2\ test$ =27.31 <30.144
0.05< pvalue<0.10 > alpha=5%
So, we fail to reject H0

Critical Method
D.R reject H0 if $\chi^2\ test > 30.144$
27.31<30.144
Therefore, do not reject H0 and there is no evidence that the standard deviation is exceeded

P3.
$H_0\ \sigma^2 = 79$

$H_a\ \sigma^2 \neq 79$ 2 sided

alpha=0.05
n=35, s=12, df=34 (use 30)

Critical Method
χ^2 crit = $\chi^2{}_{n-1,\frac{\alpha}{2}} = \chi^2{}_{34,0.025} = 46.979$ (use 30)...upper bound

χ^2 crit = $\chi^2{}_{n-1,1-\frac{\alpha}{2}} = \chi^2{}_{34,0.975} = 16.791$ (use 30)...lower bound

DR. Reject H_0 if χ^2 $test\ is > 46.979\ or < 16.791$

$\chi^2_{test} = \frac{(n-1)s^2}{\sigma^2} = \frac{34(144)}{79} = 61.97$

So, χ^2 $test\ is\ greater\ than\ 46.9792\ and\ is\ in\ the\ rejection\ region, so\ we\ reject\ H_0$

So, there is stat. evidence the variability is different than the general population

Using p-value
Go across from 34 df (use 30) and find $\chi^2_{test} = \frac{(n-1)s^2}{\sigma^2} = \frac{34(144)}{79} = 61.97$

It is greater than the last value, 53.672
So the 2 sided p-value is less than 2(0.005)
Ie. P-value <0.01 < alpha=5%
So, we reject H0

P4. $n = 27$ $H_o\ \sigma = 30$

$H_a\ \sigma < 30\ (less\ than\ test)$

$\chi^2\ critical = \chi^2\ n-1, 1-\alpha$

$= \chi^2\ 26\ , 1 - 0.05 = \chi^2\ 26{,}0.95$

$= 15.379$

Rejection region →reject $H_0\ if\ x^2 test < 15.379$

$\chi^2\ test = \frac{(n-1)s^2}{\sigma^2} = \frac{26(235)}{900} = 6.79 < \chi^2\ critical$

$\therefore reject\ H_o$ and there is evidence the standard deviation is less than 30.

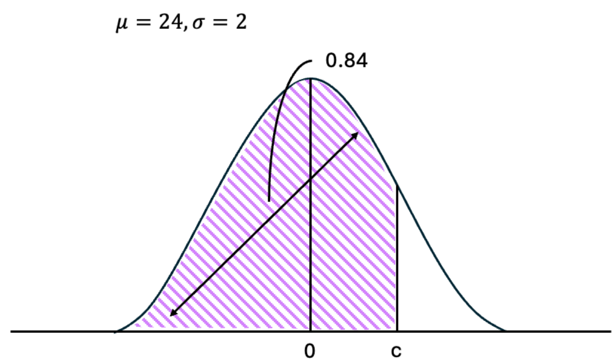
Q. Challenge Questions on Chapter 6 and 7

Q1.An automatic juice machine fills bottles with juice. Let Xbe the amount dispensed (in ounces), where $X$is normally distributed with a mean of 24 oz and a standard deviation of 2 oz.

If 84% of all bottles filled by this machine contain $c$ounces or less, find the value of c.

$$\mu = 24, \sigma = 2$$

From the standard normal table, look up the area 0.84 in the body and get: z= 1



$Z = \frac{x-\mu}{\sigma}$ cross multiply:

x=$c = \mu + z\sigma$
$c = 24 + (1)(2)$
$c = 26$

Final Answer $\boxed{26 \text{ oz}}$

Q2. Let X be a random variable that represents the amount of time (in hours) a customer spends at a coffee shop. Assume X follows an exponential distribution with a mean of 0.5 hour.

a) What is the probability the next customer spends between 15 and 30 minutes in the coffee shop?

Mean $\mu = 0.5$ hours
Rate parameter: $\lambda = \frac{1}{\mu} = 2\ customers/hour$
Convert minutes to hours:

- 15 minutes = 0.25 hours
- 30 minutes = 0.50 hours

$$P(a < X < b) = e^{-\lambda a} - e^{-\lambda b}$$

$$\begin{aligned} &P(0.25 < X < 0.50) = e^{-2(0.25)} - e^{-2(0.50)} \\ &= e^{-0.5} - e^{-1} \\ &= 0.607 - 0.368 \\ &= 0.239 \end{aligned}$$

b) What is the median time spent in the coffee shop?

The median m satisfies: Let m be the median:

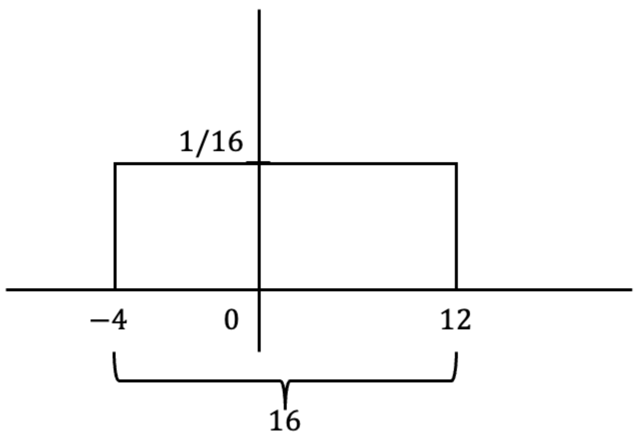
$$P(X \leq m) = 0.5$$

$$1 - e^{-\lambda a} = 0.5 \text{ and } \lambda = 2$$

$$\begin{aligned} &1 - e^{-2m} = 0.5 \\ &e^{-2m} = 0.5 \\ &-2m = \ln(0.5) \\ &m = \frac{\ln(2)}{2} \approx 0.347 \text{ hours} \\ &\approx 20.8 \text{ minutes} \end{aligned}$$

Q3. Suppose $X$is a uniform random variable on the interval $(-4,12)$. Find the lower quartile (first quartile, Q_1).

A. -1
B. 0
C. 2
D. 1
E. None of the above



For a uniform distribution on (a,b), the lower quartile is the value such that 25% of the distribution lies below it.

Step 1: Identify the interval

$$a = -4, b = 12$$

Step 2: Use the quartile formula

$$Q_1 = a + 0.25(b - a)$$

Step 3: Substitute values

$$Q_1 = -4 + 0.25(12 - (-4))$$
$$Q_1 = -4 + 0.25(16)$$
$$Q_1 = -4 + 4$$
$$Q_1 = 0$$

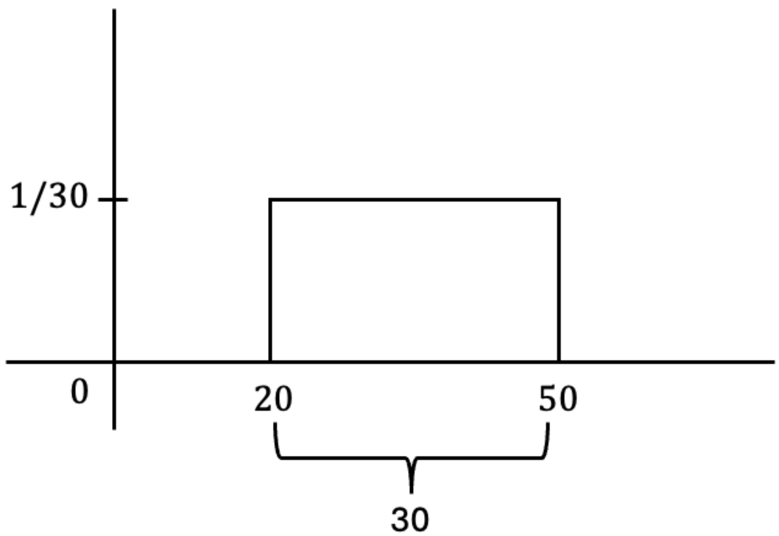
Or just do 0.25 (16) =4 and go 4 units from the left side i.e. -4 and get 0

✔ Answer choice B: 0

Q4. The time it takes to complete a load of laundry is uniformly distributed between 20 and 50 minutes. What is the probability it will take less than 35 minutes, given that it has already taken more than 25 minutes?

Let X= time (in minutes) to complete the laundry.

$$X \sim \text{Uniform}(20,50)$$

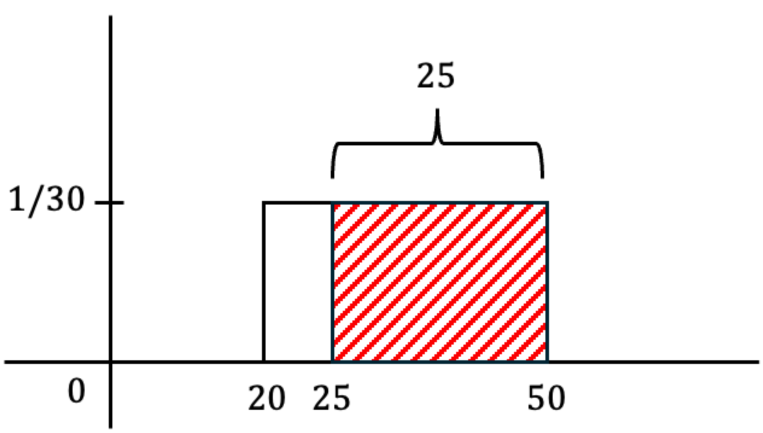


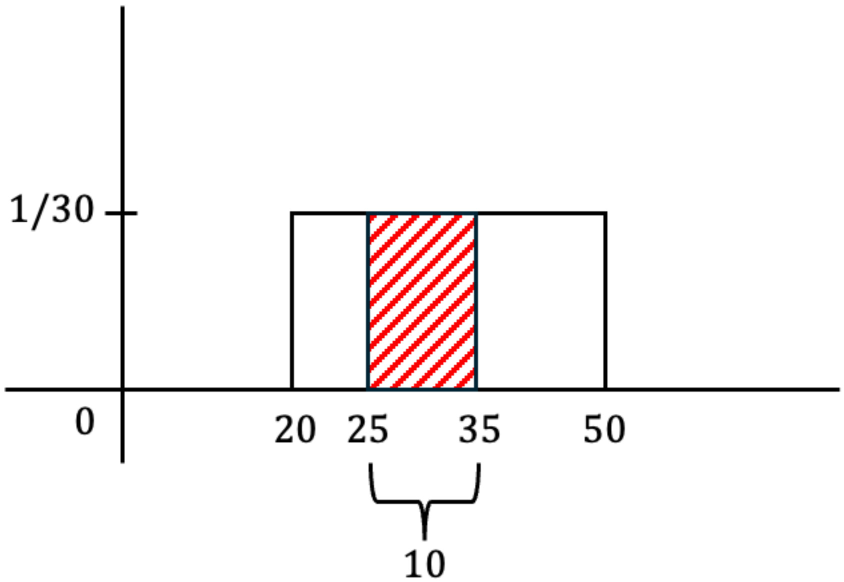
We are asked to find:

$$P(X < 35 \mid X > 25)$$

Step 1: Use the conditional probability formula: $\Pr(A/B) = \frac{\Pr(A \text{ and } B)}{\Pr(B)}$

$$P(X < 35 \mid X > 25) = \frac{Pf(X < 35 \text{ and } X > 25)}{P(X > 25)} = \frac{P(25 < X < 35)}{P(X > 25)}$$





Step 2: Find each probability using the uniform distribution

Total interval length:

$$50 - 20 = 30$$

Numerator:

$$P(25 < X < 35) = \frac{35 - 25}{30} = \frac{10}{30} = \frac{1}{3}$$

Denominator:

$$P(X > 25) = \frac{50 - 25}{30} = \frac{25}{30} = \frac{5}{6}$$

Step 3: Divide $P(X < 35 \mid X > 25) = \frac{\frac{1}{3}}{\frac{5}{6}}$

$$= \frac{1}{3} \times \frac{6}{5}$$
$$= \frac{2}{5}$$

Final Answer

$$\boxed{0.40}$$

Q5. Scores on a final exam are normally distributed with a mean of 82 and a standard deviation of 5. What test score is in the 25th percentile?

Step 1: Identify the z-score for the 25th percentile $\mu = 82 \; and \; \sigma = 5$

From the standard normal table: Look up the Area 0.25 in the body of the Z table and

we get Z= -0.67

$$z_{0.25} \approx -0.67$$

Step 2: Convert the z-score to a raw score

Use the formula: $Z = \frac{x-\mu}{\sigma}$ cross multiply

$$x = \mu + z\sigma$$
$$x = 82 + (-0.67)(5)$$
$$x = 82 - 3.35$$
$$x = 78.65$$

Final Answer $\boxed{78.7}$

A score of approximately 78.7 is at the 25th percentile, meaning about 25% of students scored below this value.

Q6. An exponential distribution has a mean of μ =40 days.

a) Find $P(X < 15)$

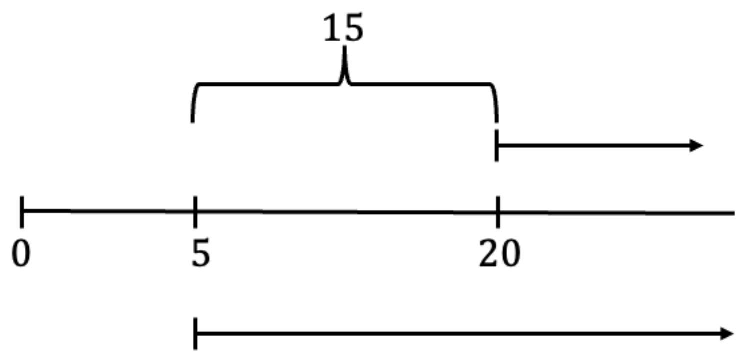
Solution $\lambda = \frac{1}{40} per \; day$

$P(X \leq a) = 1 - e^{-\lambda a}$

$$P(X < 15) = 1 - e^{-15/40}$$
$$= 1 - e^{-0.375}$$
$$= 1 - 0.687$$
$$= 0.313$$

b) Find the probability that $X \geq 20$ given $X \geq 5$

Solution (Memoryless Property)



$P(X \geq 20 \mid X \geq 5) = P(X \geq 15)$
$= e^{-\frac{15}{40}}$
$= e^{-0.375}$
$= 0.687$

Q7. The weight of a bag of flour follows a normal distribution with a mean of $\mu = 5$kg and a standard deviation of $\sigma = 1.5$kg.

If a sample of 25 bags is selected, find the probability that the sample mean weight is between 4.5 kg and 5.5 kg.

A. 0.6826
B. 0.7734
C. 0.8413
D. None of the above

μ =5, μ =1.5

Step 1: Identify the sampling distribution

For the sample mean:

$$Z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}}$$

Step 2: Convert bounds to z-scores

Lower bound: 4.5

$$z_1 = \frac{4.5 - 5}{1.5/\sqrt{25}} = \frac{-0.5}{0.3} \approx -1.67$$

Upper bound: 5.5 $z_2 = \frac{5.5-5}{\frac{1.5}{\sqrt{25}}} = \frac{0.5}{0.3} \approx 1.67$

$$\Pr(4.5 < \overline{x} < 5.5)$$

$$= \Pr(-1.67 < Z < 1.67)$$

Step 3: Find probability using standard normal

From z-tables:

$$P(Z < 1.67) \approx 0.9525$$
$$P(Z < -1.67) \approx 0.0475$$
$$P(-1.67 < Z < 1.67) = \Pr(Z < 1.67) - \Pr(Z < -1.67)$$

$$= 0.9525 - 0.0475 = 0.905$$

✅ Answer **0.905 (D. None of the above)**

Q8. At the student center on a randomly selected Friday evening, the time (in minutes) between people entering follows an exponential distribution with a $\mu = 8$ minutes.

a) If someone enters at 7:10pm, what is the probability the next person enters after 7:16pm?

$\mu = 8, so\ \lambda = \frac{1}{\mu} = \frac{1}{8} people$ per minute

$Pr(X > a) = e^{-\lambda a}$

$$P(X > 6) = e^{-6/8}$$
$$= e^{-0.75}$$
$$= 0.472$$

b) Suppose there is an 80% probability that the time between arrivals is less than $t$minutes.

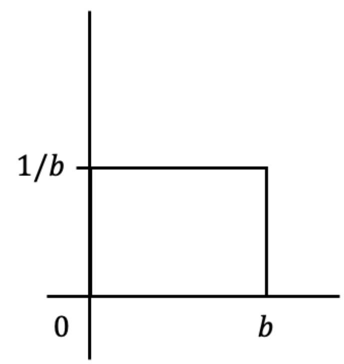
Find t. $P(X \leq a) = 1 - e^{-\lambda a}$

$$1 - e^{-t/8} = 0.80$$
$$e^{-t/8} = 0.20$$
$$-\frac{t}{8} = \ln(0.20)$$
$$t = -8\ln(0.20)$$
$$t \approx 12.9$$

Answer choice: ✔ None of the above

Q9. A uniform distribution on the interval $(0, \mathrm{b})$has a variance of 12.
Find the probability that $X > 6$.

Step 1: Use the variance formula for a uniform distribution



For $X \sim$ Uniform$(0, b)$: $\sigma^2 = 12$ and since $\sigma = \frac{b}{\sqrt{12}}$

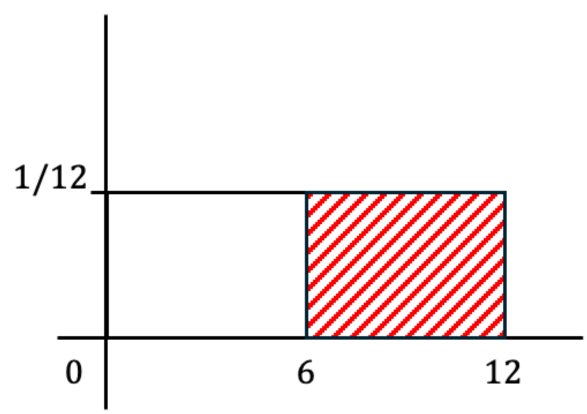
σ^2=Var$(X) = \frac{b^2}{12}$
Set this equal to 12:

$\frac{b^2}{12} = 12$ cross multiply:

Step 2: Solve for b

$b^2 = 144$
$b = 12$
Step 3: Find the probability

$$P(X > 6) = \frac{12 - 6}{12}$$
$$= \frac{6}{12}$$

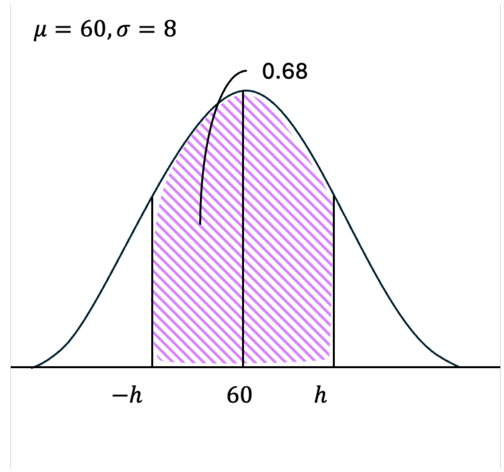


$= \frac{1}{2}$ Final Answer $\boxed{0.5}$

Q10. Heights of a certain type of shrub follow a normal distribution with a mean of 60 cm and a standard deviation of 8 cm.

1) Suppose you know that 68% of all these shrubs have heights between

$$60 \pm h$$

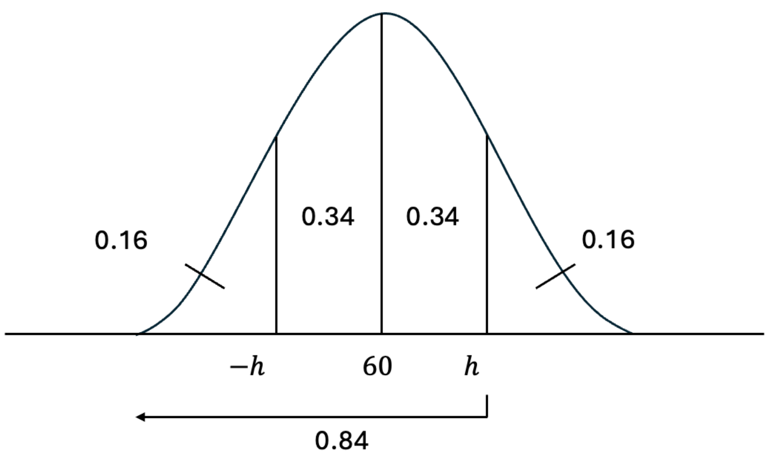


Find h. $\mu = 60$ and $\sigma = 8$

A. 4
B. 8
C. 16
D. 5.4
E. None of the above

1 – 0.68 = 0.32

So, we have 0.16 on each side and the total area below the line is 0.84



Look up AREA 0.84 in the BODY of the Z table

We get Z = 1

$$Z = \frac{x - \mu}{\sigma}$$

Cross multiply: $x = Z\,\sigma + \mu = 1(8) + 60 = 68$

So, if we have $60 + h = 68,$ $then\ the\ height\ is$ $8\ cm.$

2) Find the probability that a shrub's height is greater than 76 cm, given that it is greater than 68 cm.

A. 0.1587
B. 0.5
C. 0.0228
D. 1
E. None of the above

We are asked to find:

$$P(X > 76 \mid X > 68)$$

Step 1: Convert values to z-scores

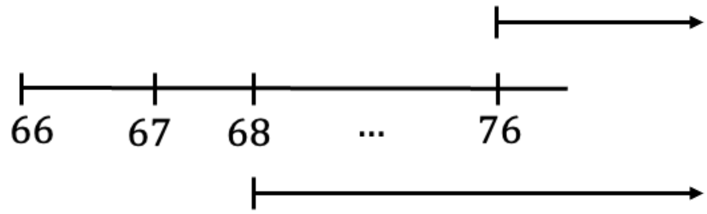
$$z_{76} = \frac{76 - 60}{8} = 2$$
$$z_{68} = \frac{68 - 60}{8} = 1$$

Step 2: Use conditional probability: Pr $(A/B) = \frac{\Pr(A\ and\ B)}{\Pr(B)}$

$$P(X > 76 \mid X > 68) = \frac{P(X > 76\ and\ X > 68)}{P(X > 68)} = \frac{P(X > 76)}{P(X > 68)}$$

Using standard normal values (no calculator):

- $P(Z > 2) = 1 - 0.9772 = 0.0228$
- $P(Z > 1) = 1 - 0.8413 = 0.1587$



Step 3: Divide

$$\frac{0.0228}{0.1587} \approx 0.144$$

✅ Correct Answer: A. 0.144

Q11. The time to fill out a visitor survey at Happy Park is normally distributed with a mean of 20 minutes and a standard deviation of 4 minutes. If 16 people fill out the survey, what is the probability that the average time per person is less than 22 minutes?

Step 1: Identify the sampling distribution

For the sample mean: $\mu = 20 \; and \; \sigma = 4$

Step 2: Convert to a z-score

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{22 - 20}{4/\sqrt{4}} = 2$$

Step 3: Find the probability

$$P(\bar{X} < 22) = P(Z < 2)$$

From standard normal tables:

$$P(Z < 2) \approx 0.9772$$

✅ Answer 0.9772

Q12. A survey shows that 40% of students own a pet. If 200 students are randomly selected, what is the probability that at least 90 of them own a pet? Use the normal approximation to the binomial.

A. 0.1587
B. 0.1611
C. 0.1711
D. 0.8289
E. None of the above

Step 1: Identify the binomial parameters and check conditions np = 200(0.4) = 80 and

nq = 200(0.6)=120 (both are >/5, so we can approximate using the normal)

$$n = 200, p = 0.40$$

We want:

$$P(X \geq 90)$$

Step 2: Find the mean and standard deviation

$$\mu = np = 200(0.4) = 80$$
$$\sigma = \sqrt{np(1-p)} = \sqrt{200(0.4)(0.6)}$$
$$\sigma = \sqrt{48} \approx 6.93)$$

Step 3: Apply continuity correction

$P(X \geq 90) \approx P(Y \geq 89.5)$ Use a number line

Step 4: Convert to standard normal z

$$z = \frac{89.5 - 80}{6.93} = 1.37$$

Step 5: Find probability from standard normal table

$$P(Z \geq 1.37) = 1 - P(Z \leq 1.37)$$

From standard z-tables:

$$P(Z \leq 1.37) \approx 0.9147$$
$$P(Z \geq 1.37) = 1 - 0.9147 \approx 0.085$$

✅ Final Answer

0.085 (None of the above, E)

Q13. The time in hours to complete a car inspection follows an exponential distribution with a mean of 6 hours.

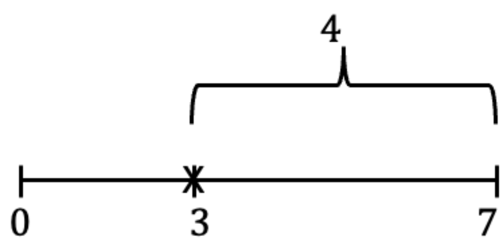
a) What is the probability that an inspection lasts more than 4 hours?

$\mu = 6\ hours, so\ \lambda = \frac{1}{6}$ cars per hour

$$P(X > 4) = e^{-4/6}$$
$$= e^{-0.667}$$
$$= 0.513$$

✔ None of the above

b) What is the conditional probability that an inspection lasts less than 7 hours, given that it lasts more than 3 hours?

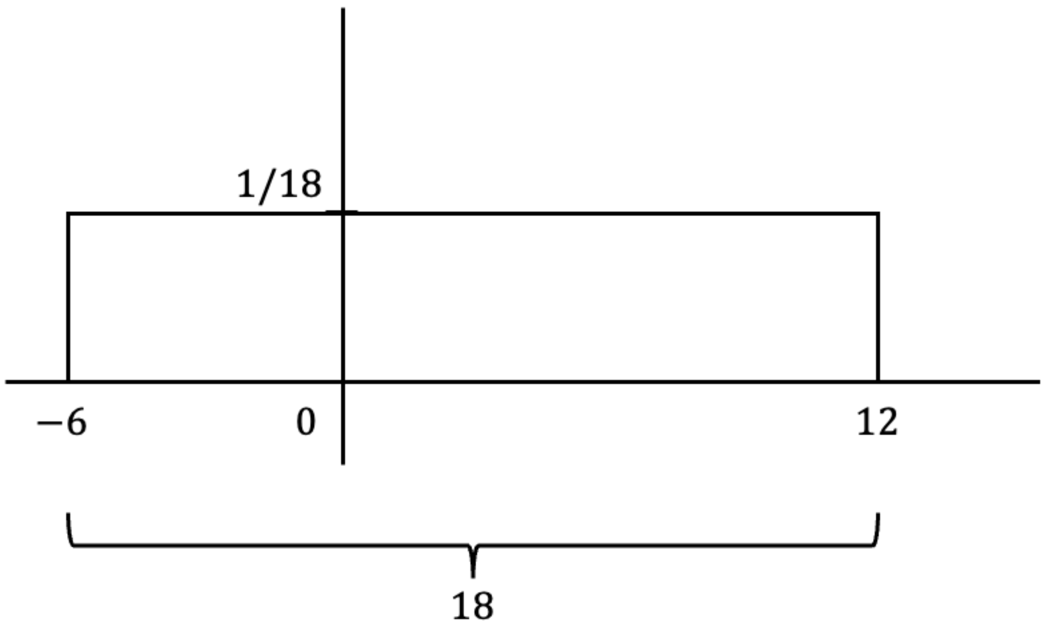


Solution $P(X < 7 \mid X > 3) = 1 - P(X \geq 7 \mid X \geq 3)$
Using the memoryless property: It doesn't know you've been waiting 3 hours, so if it is in less than 7, that is the same as in the next 4 hours.

$Pr(X < a) = 1 - e^{-\lambda a}$

$$P(X \leq 4)$$
$$= 1 - e^{-4/6}$$
$$= 1 - 0.513 = 0.487$$

Q14. Suppose a random variable X follows a uniform distribution on $(-6,12)$.
What is the probability that $X$is greater than 2, given that $X$is less than 8?

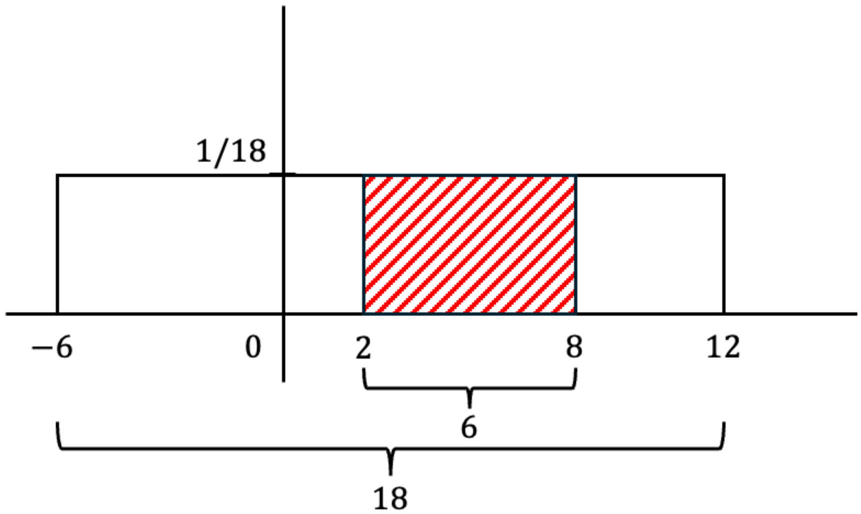


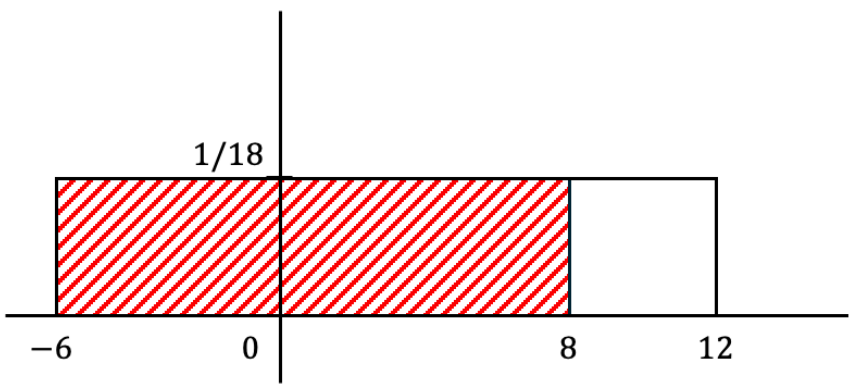
We are asked to find:

$$P(X > 2 \mid X < 8)$$

Step 1: Use the conditional probability formula: Pr $(A/B) = \frac{\Pr(A \text{ and } B)}{\Pr(B)}$

$$P(X > 2 \mid X < 8) = \frac{\Pr(X > 2 \text{ and } X < 8)}{P(X < 8)} = \frac{P(2 < X < 8)}{P(X < 8)}$$





Step 2: Find each probability using the uniform distribution

Total interval length:

$$12-(-6)=18$$

Numerator: $P(2<X<8)=\frac{8-2}{18}=\frac{6}{18}=\frac{1}{3}$

Denominator: $P(X<8)=\frac{8-(-6)}{18}=\frac{14}{18}=\frac{7}{9}$

Step 3: Divide $P(X > 2 \mid X < 8) = \frac{\frac{1}{3}}{\frac{7}{9}}$

$$= \frac{1}{3} \times \frac{9}{7}$$
$$= \frac{3}{7}$$

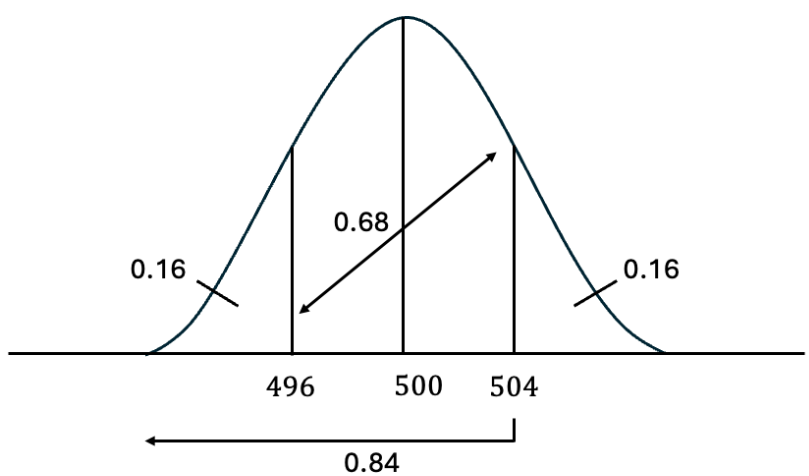
Final Answer $\boxed{\frac{3}{7}}$

Q15. The weight of cereal in boxes is normally distributed with a mean of 500 grams. It is known that 68% of the boxes have weights between 496 g and 504 g.

Find the standard deviation of the cereal box weights.

A. 2
B. 4
C. 6
D. 8
E. None of the above

$\mu = 500\ and\ \sigma = ?$ We have 1 – 0.68= 0.32 and 0.32/2 = 0.16



Look up AREA in the body of the Z table and get Z = 1

$$Z = \frac{x - \mu}{\sigma}$$

$$1 = \frac{504 - 500}{\sigma}$$

Cross multiply, and get $\sigma = 4$

Q16. In a study of diabetes, there are 4000 patients:

- 800 are under 20 years old
- 1200 are 20–39 years old
- 2000 are 40 and older

You want to take a stratified sample of 160 patients where the strata are the age groups.

How many respondents should be selected from each stratum?

Solution Step 1: Find the proportion of each stratum

$$\text{Proportion under 20} = \frac{800}{4000} = 0.2$$
$$\text{Proportion 20–39} = \frac{1200}{4000} = 0.3$$
$$\text{Proportion 40+} = \frac{2000}{4000} = 0.5$$

Step 2: Multiply each proportion by the total sample size

$$n_{\text{under 20}} = 0.2 \times 160 = 32$$
$$n_{20\text{–}39} = 0.3 \times 160 = 48$$
$$n_{40+} = 0.5 \times 160 = 80$$

Age Group	Sample Size
Under 20	32
20–39	48
40+	80

Q17. Suppose you have a population of 900 first-year engineering students, and it is known that the number of students checking Instagram on a certain day has a mean of $\mu = 15$ and a standard deviation of $\sigma = 6$. You take a random sample of 64 students from this population.

What is the probability that the sample mean number of students checking Instagram exceeds 16?

A. 0.1587 B. 0.8413 C. 0.3085 D. 0.6915 E. None of the above

$\frac{n}{N} = \frac{64}{900} = 0.07 > 0.05$ So, we need to use the Finite Correction Factor

$$\mu = 15, n = 64, \sigma = 6$$

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}}$$

$$\sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{900-64}{899}} = 0.964324704$$

$$P(\bar{X} > 16) = \Pr(> \frac{16-15}{6/\sqrt{64}(0.964324704)}) = \Pr(Z > 1.38) = 1 - \Pr(Z < 1.38)$$

$$P(Z < 1.38) \approx 1 - 0.9162 = 0.0838$$

✅ Answer 0.0838 (E. None of the above)

R. Challenge Questions on Chapter 8 and 9

Answer each of the following on a separate page. Do NOT look at your notes and see how much you can remember!

R1. The true population proportion is 35% for the number of adults who exercise at least 3 times per week. A random sample of 900 adults was performed, and the margin of error was 3%.

What level of confidence was used?

A. 90%
B. 95%
C. 99%
D. 92%

Solution

Step 1: Recall the margin of error formula for a proportion

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{p(1-p)}{n}}$$

Where:

- $E = 0.03$
- $p = 0.35$
- $n = 900$

We need to solve for $Z_{\alpha/2}$.

Step 2:

$$0.03 = Z_{\alpha/2} \cdot \sqrt{\frac{0.35(0.65)}{900}}$$

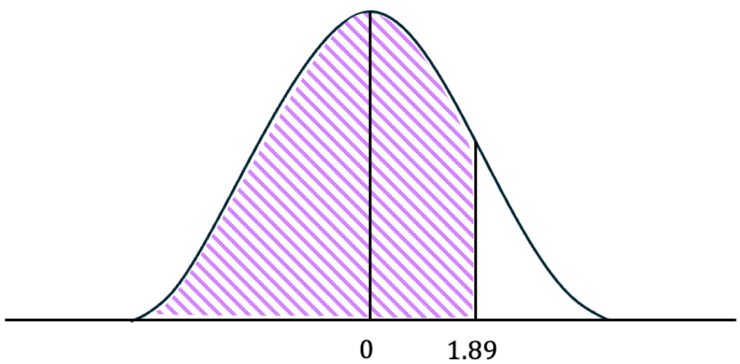
Step 3: Solve for $Z_{\alpha/2}$

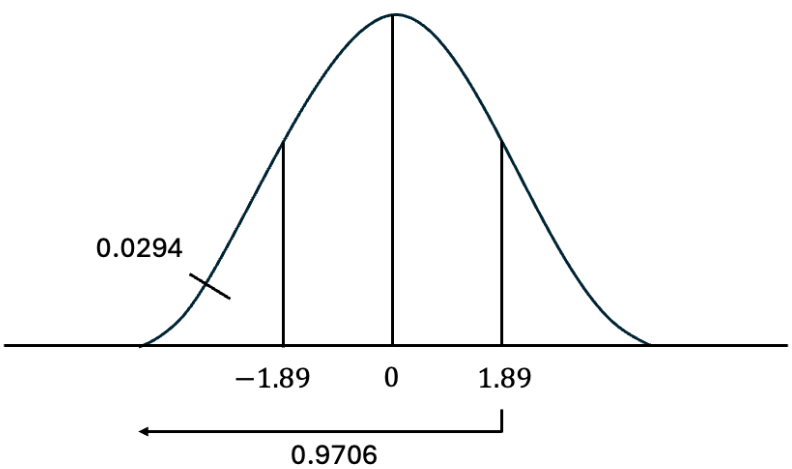
$$Z_{\alpha/2} = \frac{E}{\sigma_{\hat{p}}} = \frac{0.03}{0.0159} \approx 1.89$$

Step 4: Find the confidence level

- $Z_{\alpha/2} \approx 1.89$
- From standard normal tables:
  - $P(-1.89 < Z < 1.89) \approx 0.9706 - 0.0294 = 0.9412$

So, the confidence level is 94% (closest to 95% in the multiple choice options).





✅ Answer

B. 95%

R2. A nutrition research group wishes to estimate the mean number of servings of fruits and vegetables consumed per week by members of a community. They want to estimate the mean within 1 serving with a 95% confidence level. Previous studies suggest the standard deviation is 3 servings.

Which of the following is the smallest sample size that meets these criteria?

A. 9 B. 35 C. 35.3 D. 36 E. None of the above

Solution

Step 1: Recall the sample size formula

For estimating a mean with known σ:

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

Where:

- $z_{\alpha/2}$= z-value for the desired confidence level
- σ= population standard deviation
- E= desired margin of error

Step 2: Identify the values

- 95% confidence → $z_{\alpha/2} = 1.96$
- $\sigma = 3$
- $E = 1$ (within one serving)

Step 3: Plug into the formula

$$n = \left(\frac{1.96 \cdot 3}{1}\right)^2$$
$$n = (5.88)^2 \approx 34.5744$$

Step 4: Round up $n = 35$

✅ Answer 35 (B)

R3. At a fast-food restaurant, management wants customers to wait no more than 4 minutes to receive their order at the counter. Past records show the waiting time is strongly skewed to the right with a standard deviation of 1.6 minutes.

To test whether the mean waiting time is less than 4 minutes, management takes a simple random sample of 50 customers. The sample mean waiting time is 3.6 minutes.

Test the hypotheses at the $\alpha = 0.05$ significance level.

Step 1: State the Hypotheses

$H_0: \mu = 4$
$H_a: \mu < 4$ (1-sided)

$$\sigma = 1.6\ (use\ Z), n = 50, \bar{x} = 3.6$$

(This is a left-tailed test because management wants to know if the mean wait time is *less than* 4 minutes.)

Step 2: Check Conditions

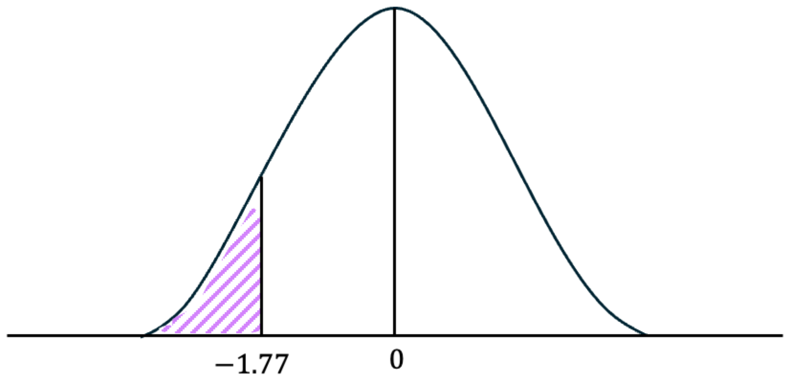
- SRS given ✓
- Population is skewed, but $n = 50 \geq 30 \rightarrow$ CLT applies ✓
- Standard deviation is known → use z-test ✓

Step 3: Test Statistic

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$
$$z = \frac{3.6 - 4}{1.6/\sqrt{50}}$$
$$z = \frac{-0.4}{0.226}$$
$$z \approx -1.77$$

✅ P-Value Method

Step 4: Find the P-Value



For a left-tailed test:

$$P(z < -1.77) \approx 0.038$$

Step 5: Decision (P-Value Method)

- p-value $= 0.038$
- $\alpha = 0.05$

Since: $p - value = 0.038 < 0.05 = \alpha$
👉 Reject H_0

Step 6: Conclusion (P-Value Method)

There is statistically significant evidence that the mean customer waiting time is less than 4 minutes.

Correct Conclusion Choice

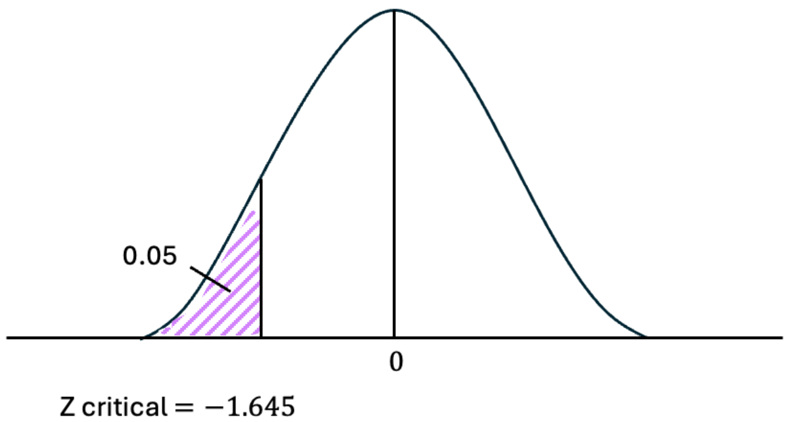
✔ The value 3.6 is a statistically significant result

✅ Critical-Value (Critical Method)

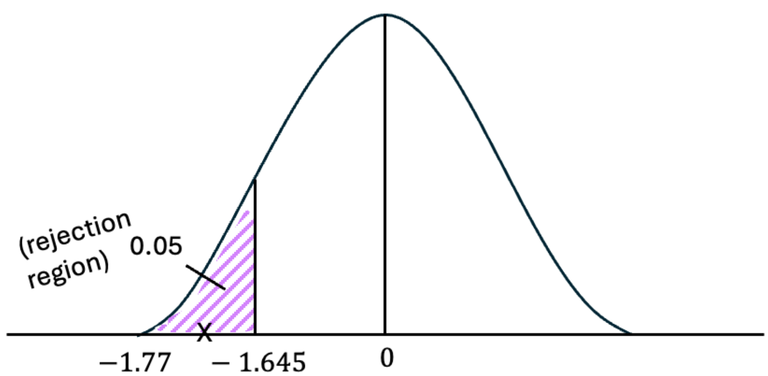
Step 7: Find the Critical Value

For a left-tailed test with $\alpha = 0.05$:

$$z_{\text{critical}} = -1.645$$



Step 8: Compare Test Statistic to Critical Value



- Test statistic: $z = -1.77$
- Critical value: $z = -1.645$

Since: $-1.77 < -1.645$ 👉 Reject H_0

R4. A manufacturer measures the breaking strength of steel rods. The population standard deviation is known to be σ = 12 MPa. A 98% confidence interval for the mean breaking strength is calculated to be:

(145.2,178.8) MPa.

Find the 90% confidence interval for the mean breaking strength.

Solution Step 1: Identify the sample mean and margin of error from the given CI

The confidence interval formula is:

$\text{CI} = \bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ where $\sigma = 12$

The sample mean $\bar{X}$is the midpoint of the CI:

$$\bar{X} = \frac{145.2 + 178.8}{2} = \frac{324}{2} = 162$$

2. The margin of error (E) for the 98% CI is half the width:

$$E_{98\%} = \frac{178.8 - 145.2}{2} = \frac{33.6}{2} = 16.8$$

Step 2: Find the standard error

For a 98% CI:

The z-value for 98% CI ($\alpha = 0.02$) is $z_{0.01} \approx 2.33$.

Step 3: Find the new margin of error for 90% CI

For 90% CI: $z_{0.05} \approx 1.645$

$new\ E_{90\%} = old\ E\ \div old\ z_{0.01} \times new\ z_{0.05}$ $=16.8\div 2.33 \times 1.645 = 11.86$

Step 4: Construct the 90% CI

$$\text{CI}_{90\%} = \bar{X} \pm E_{90\%} = 162 \pm 11.85$$
$$\text{CI}_{90\%} = (150.15,173.85)$$

✅ Answer

$$\boxed{(150.14, 173.86)}$$

R5. A simple random sample of 40 tutors indicates that the average monthly amount spent on books for their students is \$240, with a process standard deviation of \$50. Jordan believes the actual mean spending is more than \$220. Assume spending is normally distributed.

a) How much evidence is there against the null hypothesis

$$H_0: \mu = 220\,?$$

- Some (mild) evidence against H_0
- Strong evidence against H_0
- Very strong evidence against H_0
- Extremely strong evidence against H_0

b) What conclusion would you make at a 10% significance level?

A. \$220 is a plausible value for μ; \$240 is statistically significant
B. \$220 is a plausible value for μ; \$240 is not statistically significant
C. \$220 is not a plausible value for μ; \$240 is statistically significant
D. \$220 is not a plausible value for μ; \$240 is not statistically significant

Solution Step 1: Identify the sample statistics

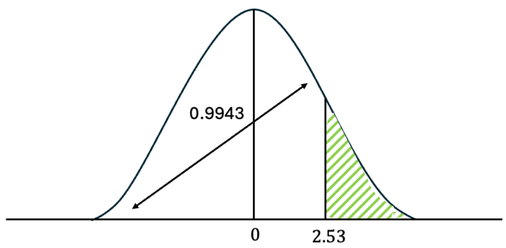
- Sample mean: $\bar{X} = 240$
- Population standard deviation: $\sigma = 50$ (process standard deviation, so use Z)
- Sample size: $n = 40$
- $H_0: \mu = 220$

We are testing a one-sided hypothesis: $H_a: \mu > 220$ (1-sided)

Step 2: Compute the test statistic Z

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

$$Z = \frac{240 - 220}{\frac{50}{\sqrt{40}}} \approx \frac{20}{7.905} \approx 2.53$$

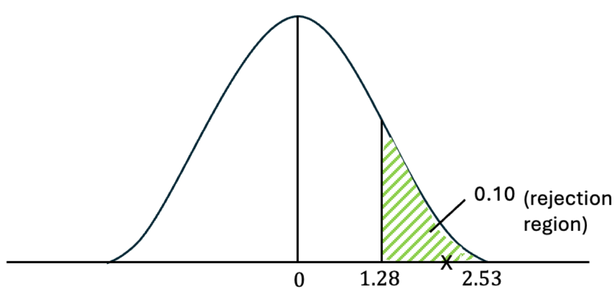


Step 3: Interpret the Z-value

Using the p-value, the p-value = 1- 0.9943 = 0.0057 < 0.10 so we reject H0

Using the critical method:

- For a one-sided test at 10% significance: critical $Z_{0.10} \approx 1.28$
- Calculated $Z = 2.53 > 1.28$→ significant



Evidence against H_0:

- Z = 2.53 corresponds to a p-value ≈ 0.006 → very small.
- So, there is very strong evidence against H_0

✅ Answer (a): Very strong evidence against H_0

Step 4: Make conclusion at 10% significance

- Since $Z = 2.53 > Z_{0.10} = 1.28$, we reject H_0.
- Therefore, $220 is not a plausible value for μ, and $240 is statistically significant.

✅ Answer (b): C. $220 is not a plausible value for μ; $240 is statistically significant

- For one-sided tests, compare Z to $Z_\alpha$instead of $Z_{\alpha/2}$.
- Small p-value → strong evidence against H_0.

R6. Some researchers are concerned that drivers on a city highway are too inconsistent in their speeds. They claim that if the standard deviation of speeds exceeds 25 km/h, the risk of accidents increases significantly.

You take a random sample of 16 vehicles and measure their speeds. The sample standard deviation is 31 km/h.

Perform a chi-square test for the population standard deviation at α = 0.10. Use:

$H_0: \sigma = 25$ vs .$H_a: \sigma > 25$ (standard deviation, not a mean, so it is a Chi Square test)

Which is the correct test statistic and decision?

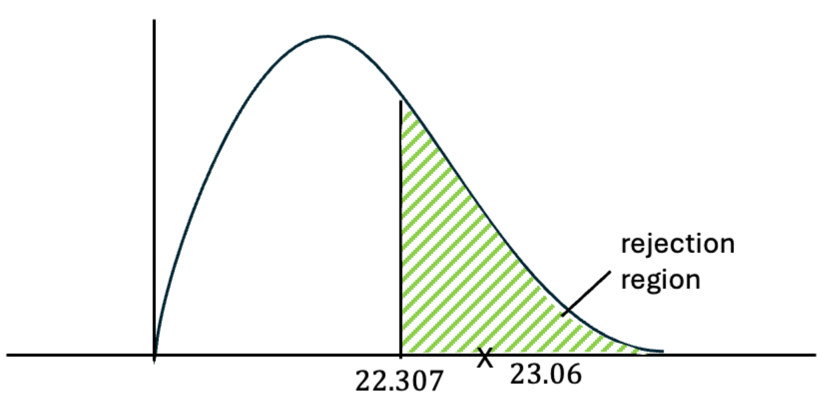
A. 24.8; reject H_0
B. 24.8; do not reject H_0
C. 30.72; reject H_0
D. 30.72; do not reject H_0

n=16 and s=31, df=16-1 = 15

Step 1: Test statistic formula

For variance (σ^2) testing:

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$
$$n = 16, s = 31, \sigma_0 = 25$$
$$s^2 = 31^2 = 961, \sigma_0^2 = 25^2 = 625$$
$$\chi^2 = \frac{(16-1)(961)}{625} = \frac{15 \cdot 961}{625} = \frac{14{,}415}{625} \approx 23.06$$



Step 2: Determine critical value

- Right-tailed test at α = 0.10
- Degrees of freedom: df = n-1 = 15
- From chi-square table: $\chi^2_{0.90,15} \approx 22.307$

Decision rule: Reject H_0if $\chi^2 > 22.307$

Step 3: Compare

$\chi^2 = 23.06 > 22.307 \implies$ Reject H_0 i.e. 23.06 is in the rejection region
✅ Answer: C. 30.72; reject H_0

R7. It is known that 20% of the population prefers tea over coffee. In a sample of 200 people, the probability that the sample proportion is less than m is 0.1587.

Find the value of m.

Solution

Step 1: Identify the parameters

- Population proportion: $p = 0.20$
- Sample size: $n = 200$

Step 2: Find the Z-score corresponding to the probability

- $P(\hat{p} < m) = 0.1587$
- From standard normal table: $P(Z < -1) = 0.1587$ (look up the AREA 0.1587 in the BODY of the Z table and we get Z= -1)
- So $Z = -1$

Step 3: solve for m

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(q_0)}{n}}}$$

$$-1 = \frac{m - 0.2}{\sqrt{\frac{0.2 \cdot 0.8}{200}}}$$

$$-1 = \frac{m - 0.2}{0.0283}$$

$$m - 0.2 = -0.0283$$

$$m = 0.1717 \approx 0.172$$

✅ Answer $\boxed{m \approx 0.172}$

R8.A survey indicates that the average cost of a new electric bicycle is $2,400. Sarah believes the true average cost is actually higher. She takes a random sample of 12 people who recently bought electric bicycles and finds the sample mean is $2,750 with a sample standard deviation of $400. Assume the costs are normally distributed.

Find:

a) The hypotheses

b) The test statistic

c) The correct decision at the 1%, 5%, and 10% levels of significance

d) Suppose your decision was not to reject H_0. What would your conclusion?

A. $2,400 is a plausible value for μ; x̄ = $2,750 is statistically significant
B. $2,400 is a plausible value for μ; x̄ = $2,750 is statistically insignificant
C. $2,400 is not a plausible value for μ; x̄ = $2,750 is statistically significant
D. $2,400 is not a plausible value for μ; x̄ = $2,750 is statistically insignificant

n=12, s=400 (use t) and $\bar{x} = 2750$

Step a) Hypotheses

$$H_0: \mu = 2400 \text{ vs. } H_a: \mu > 2400$$

One-tailed test (Sarah believes the mean is higher).

Step b) Test statistic

Since $\sigma$is unknown use the t-test:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$t = \frac{2750 - 2400}{400/\sqrt{12}} = \frac{350}{400/3.464} = \frac{350}{115.47} \approx 3.03$$

- Degrees of freedom: $df = n - 1 = 11$

Step c) Decision at different significance levels

- t-critical values (one-tailed) from t-table for df = 11:
    - 10% → $t_{0.10} \approx 1.363$
    - 5% → $t_{0.05} \approx 1.796$
    - 1% → $t_{0.01} \approx 2.718$
- Compare $t_{\text{calc}} = 3.03$ to critical values:

α	t-critical	Decision
10%	1.363	Reject H0
5%	1.796	Reject H0
1%	2.718	Reject H0

✅ We reject H0 at all three levels.

Step d) Conclusion

Since we rejected H_0, the sample provides strong evidence that the true mean cost is greater than $2,400.

- Correct choice:

C. $2,400 is not a plausible value for μ; x̄ = $2,750 is statistically significant

R9. A bakery wants to estimate how many cookies a customer buys per week. They wish to construct a 95% confidence interval for the mean number of cookies bought, with a width of no more than 6 cookies.

Based on past experience, they expect the number of cookies bought to range from 10 to 50 cookies.

How many customers do they need to sample?

Step 1: Estimate σ using the range Rule of thumb:

$$\sigma \approx \frac{\text{range}}{4}$$
$$\sigma \approx \frac{50-10}{4} = \frac{40}{4} = 10$$

Step 2: Recall the sample size formula

For a confidence interval with known σ:

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

Where: $z_{\alpha/2}$= z-value for confidence level

- $\sigma = 10$
- $E = \text{width}/2 = \frac{6}{2} = 3$

Step 3: Find $z_{\alpha/2}$

95% confidence → $z_{\alpha/2} \approx 1.96$

Step 4: Plug into the formula

$$n = \left(\frac{1.96 \cdot 10}{3}\right)^2$$
$$n = \left(\frac{19.6}{3}\right)^2 \approx (6.533)^2 \approx 42.7$$

Step 5: Round up

$$n = 43 \text{ customers}$$

R10. A random sample of 50 items is selected from a population of 300 items. The sample mean is 125, and the population standard deviation is 30.

Construct a 90% confidence interval to estimate the population mean/

Since n/N = 50/300 = 0.167 > 0.05, we use the finite correction factor

Step 1: Identify the formula with finite population correction

The standard formula for a confidence interval with known σ and FPC is:

$$CI = \bar{X} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}$$

Where:

- $\bar{X} = 125$
- $\sigma = 30$
- $n = 50, N = 300$
- 90% CI → $z_{0.05} \approx 1.645$

Step 2: $\sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{300-50}{300-1}} = \sqrt{\frac{250}{299}} = 0.914396196$

Step 3:

$$\mu = 125 \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$

$$\mu = 125 \pm 1.645 \frac{30}{\sqrt{50}} (0.914396196)$$

$$\mu = 125 \pm 6.38 = (118.62, 131.38)$$

R11. A 95% confidence interval for the population mean μ, with σ known (use Z), was determined to be:

$$(44.6, 55.4)$$

This was based on a sample size of 36 drawn from a normal population.

Find the population standard deviation σ.

Solution Step 1: Recall the CI formula

For a known σ:

$$\mu = \bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$\bar{X}$= sample mean

- $n = 36$
- 95% CI → $z_{\alpha/2} = 1.96$

Step 2: Find the margin of error

$E =$ half the width of CI $= \dfrac{55.4 - 44.6}{2} = \dfrac{10.8}{2} = 5.4$

Step 3: $E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$5.4 = 1.96(\frac{\sigma}{\sqrt{36}})$

$5.4 = 0.32666\sigma$

$\sigma =\approx 16.53$

R12. A 95% confidence interval for the mean weekly study hours of college students is:

$(12.5, 18.3)$. A pizza delivery company wants to check if its delivery times are consistent. The company standard is: $H_0: \sigma^2 = 9$ vs $H_a: \sigma^2 > 9$ (Since it is a variance and not a mean, it is a Chi Square test)

a) Find the test statistic

A. 27.25
B. 26.88
C. 27.0
D. 3.5
E. None of the above

b) Find the critical value

A. 30.14
B. 31.41
C. 32.85
D. 29.62
E. None of the above

c) Suppose your decision is "Do not reject H_0". What is your conclusion?

A. Statistically significant; meeting company standard
B. Statistically significant; not meeting company standard
C. Statistically insignificant; meeting company standard
D. Statistically insignificant; not meeting company standard

Step 1: s=3.5 n=20, $\sigma^2 = 9, \alpha = 0.05$

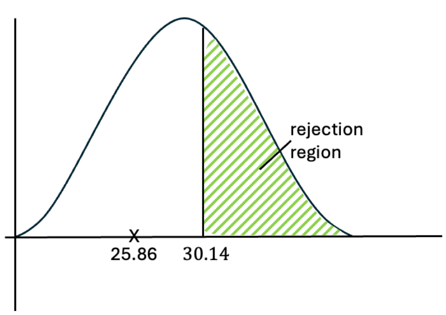
Test statistic

Chi-square formula:

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$
$$n = 20, s^2 = 3.5^2 = 12.25, \sigma_0^2 = 9$$
$$\chi^2 = \frac{(20-1)(12.25)}{9} = \frac{19 \cdot 12.25}{9} = \frac{232.75}{9} \approx 25.86$$

✅ Answer (a): E. None of the above (closest value is 25.86)

Step 2: Critical value



Right-tailed test, α = 0.05, df = n-1 = 19

From chi-square table: $\chi^2_{0.95,19} \approx 30.14$

✅ Answer (b): A. 30.14

Step 3: Compare test statistic with critical value

$\chi^2 = 25.86 < 30.14 \implies$ Do not reject H_0 (it is not in the rejection region)

✅ Answer (c): C. Statistically insignificant; meeting company standard

R13. Consider the following hypotheses and the 90% confidence interval (12.5, 18.3).

I. $H_0: \mu = 15$ vs $H_a: \mu \neq 15$
II. $H_0: \mu = 20$ vs $H_a: \mu \neq 20$

Which of these null hypotheses would be rejected at the 5% level of significance?

A. You would reject H_0 in both I and II.
B. You would reject H_0 in I, but not in II.
C. You would reject H_0 in II, but not in I.
D. You would not reject H_0 in either I or II.

Solution Step 1: Recall the CI → hypothesis test relationship

- For a two-tailed test at significance level α:
  - If μ_0 is inside the CI, fail to reject H_0.
  - If μ_0 is outside the CI, reject H_0.
- 95% CI corresponds to a 5% significance level for a two-tailed test.

Step 2: Check each null hypothesis

- CI: (12.5, 18.3)

I. $H_0: \mu = 15$

- 15 is inside the CI → fail to reject H_0

II. $H_0: \mu = 20$

- 20 is outside the CI → reject H_0

✅ Answer

C. You would reject H_0 in II, but not in I.

R14. A random sample of 18 students found that the average amount spent on school supplies was \$63 with a sample standard deviation of \$15.

You wish to test the hypotheses:

$H_0: \mu = 55$ vs.$H_a: \mu \neq 55$ (2-sided test)

at the 5% level of significance.

a) What are the correct critical values to use in your rejection region(s)?

b) Which of the following is the correct decision and conclusion?

A. Do not reject H_0; \$55 is a plausible value for μ
B. Do not reject H_0; \$55 is not a plausible value for μ
C. Reject H_0; \$55 is a plausible value for μ
D. Reject H_0; \$55 is not a plausible value for μ

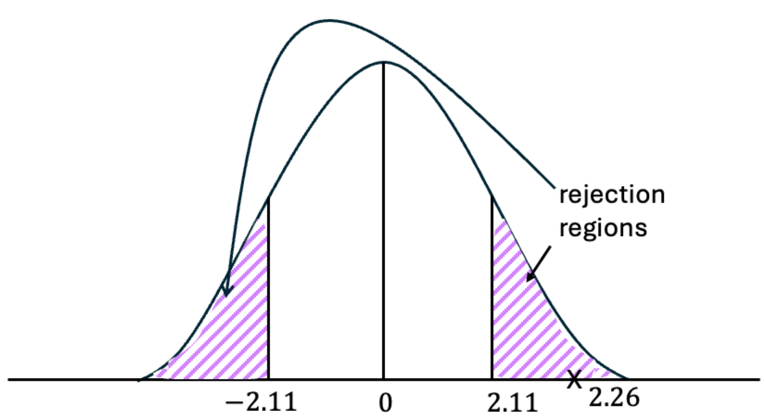
c) Answer parts a) and b) again given your alternative hypothesis is now a "greater than" alternative.

n=18, s=15 (use t), $\alpha = 0.05, \bar{x} = 63$

Step 1: Compute the t-statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{63 - 55}{15/\sqrt{18}} = \frac{8}{15/4.243} = \frac{8}{3.536} \approx 2.26$$

- Degrees of freedom: $df = n - 1 = 17$



Step 2: Find critical t-values (two-tailed, α = 0.05, df = 17)

- From t-table: $t_{0.025,17} \approx \pm 2.11$

Step 3: Decision

- $|\, t \,| = 2.26 > 2.11 \implies$ Reject H_0 (2.26 is in the rejection region)

✅ Answer (two-tailed): D. Reject H_0; \$55 is not a plausible value for μ

- For one-tailed (Ha: μ > 55), t-critical = 1.74
- $t = 2.26 > 1.74 \implies$ Reject H0

✅ Answer (one-tailed): Reject H0

R15. A confidence interval for μ was constructed, where σ is unknown for a normal population. The interval was (48.12, 63.88), based on a sample size of 25 with a sample variance of 64.

What level of confidence was used?

A. 90%
B. 95%
C. 99%
D. 97%

σ is unknown (t test), n=25, s^2=64

Step 1: Compute the sample standard deviation

$$s = \sqrt{64} = 8$$

Step 2: Find the sample mean and margin of error

- Margin of error: $E = \frac{width}{2} = \frac{63.88-48.12}{2} = 7.88$

Step 3: Find t-value used

$$E = t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$7.88 = t_{\frac{\alpha}{2}}\left(\frac{8}{\sqrt{25}}\right)$$

$$t_{\frac{\alpha}{2}} = \frac{7.88}{1.6} \approx 4.925$$

Degrees of freedom: $df = n - 1 = 24$

- From t-table: $t_{0.005,24} \approx 2.797$, $t_{0.0025,24} \approx 3.745$, $t_{0.001,24} \approx 4.24$
- Closest: $t \approx 4.925$→ 99% confidence level

✅ Answer: C. 99%

R16. Maya measured the time it takes to assemble a bicycle wheel. It is known that the population standard deviation is σ = 8 minutes. She buys a new set of tools hoping to reduce the variability. She takes a random sample of 12 assembly times and finds a sample standard deviation of s = 6 minutes.

Perform the appropriate test for the population variance at the 5% level of significance. Assume the time to assemble wheels is normally distributed.

A. 6.75; reject H_0
B. 6.75; fail to reject H_0
C. 1.44; reject H_0
D. 1.44; fail to reject H_0

n=12, s=6, $\alpha = 0.05, \sigma = 8$

Step 1: State hypotheses

- $H_0: \sigma^2 = 64$(σ = 8 → σ² = 64) Since it is about variance, it is a Chi square test
- $H_a: \sigma^2 < 64$ (she hopes to reduce variability, so it is a 1-sided test)

Step 2: Test statistic

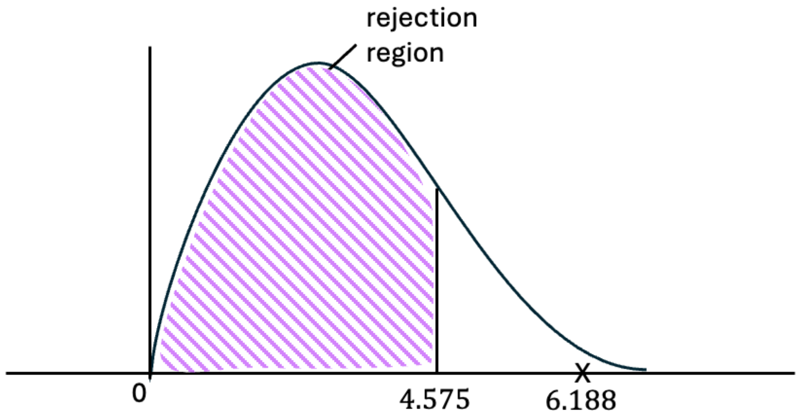
The chi-square test statistic for variance:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$
$$n = 12, s^2 = 6^2 = 36, \sigma^2 = 64$$
$$\chi^2 = \frac{(12-1)(36)}{64} = \frac{11 \cdot 36}{64} = \frac{396}{64} \approx 6.1875$$

Step 3: Critical value

- Left-tailed test at α = 0.05, df = n-1 = 11
- From chi-square table: $\chi^2_{0.05,11} \approx 4.575$

- Decision rule: reject H_0 if $\chi^2 < 4.575$



Step 4: Compare test statistic

$\chi^2 = 6.188 > 4.575 \implies$ fail to reject H_0 since it is in the rejection region

✅ Answer: B. 6.75; fail to reject H_0(slight rounding difference for multiple-choice)

R17. Given:

$H_0: \mu = 120$ vs. $H_a: \mu > 120$ (a 1-sided test)
You test at α = 0.05 and find that the true mean is μ = 130. If the probability of making a Type II error (β) is 0.10, what is the probability that you will fail to reject H_0?

A. 0.90
B. 0.10
C. 0.05
D. 0.95
E. None of the above

Solution By definition, Type II error (β) is the probability of not rejecting H_0 when H_a is true.

- Here, H_a is true (μ = 130) and β = 0.10.
- Therefore, the probability of not rejecting H_0 = β = 0.10

✅ Answer: B. 0.10

R18.Which of the following statements is true?

I) A Type I error occurs when we reject H_0 and H_0 is actually true.
II) If the null hypothesis is rejected at α = 0.01, it will also be rejected at α = 0.05.

A. I only
B. II only
C. Both I and II
D. Neither I nor II

Solution Type I error occurs when we reject H_0 when it is true → statement I is true

- If H_0 is rejected at α = 0.01, the p-value < 0.01 → it is also < 0.05 → so H_0 will also be rejected at α = 0.05 → statement II is true.

✅ Answer: C. Both are true

R19.Suppose at a company, employees are surveyed about whether they prefer flexible work hours. Approximately 60% of all employees prefer flexible hours. You take a random sample of 100 employees, and the probability that more than k% of the sample prefer flexible hours is 10%.

What is the value of k?

n=100, p_0=0.60

Step 1: Identify the parameters

- Population proportion: $p_0 = 0.60$
- Sample size: $n = 100$

Step 2: Find the Z-score corresponding to the probability

- Probability $P(\hat{p} > k) = 0.10$
- So $P(\hat{p} < k) = 0.90$
- From the standard normal table: $P(Z < 1.28) \approx 0.90$ (look up the area below the line 0.90 in the BODY of the Z table and get Z=1.28)
- Therefore, $Z = 1.28$

Step 3: solve for k $$Z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(q_0)}{n}}}$$

$$1.28 = \frac{k - 0.6}{\sqrt{\frac{0.6 \cdot 0.4}{100}}}$$

$$1.28 = \frac{m - 0.6}{0.048989794}$$

$$m - 0.6 = 0.0627$$

✅ Answer $\boxed{m \approx 0.6627}$
$\boxed{k \approx 66.3\%}$

R20. Suppose we want to test:

$H_0: \mu = 50$ vs $H_a: \mu \neq 50$ (2-sided test)
A 95% confidence interval for $\mu$is:

$$(51.2, 59.8)$$

Using this confidence interval, which conclusion is correct?

- A. We reject H_0at both 5% and 10% levels of significance
- B. We cannot reject at both 5% and 10% levels of significance
- C. We cannot reject at 5% level but we would likely reject H_0at the 10% level of significance
- D. We cannot reject at 5% level but we are unsure if we reject or not at the 10% level of significance

Solution Step 1: Recall the CI → Hypothesis Test relationship

- A two-sided hypothesis test at significance level αcan be tested using a $(1 - \alpha)$ CI:

  If μ_0 is inside the CI, we fail to reject H_0
  If μ_0 is outside the CI, we reject H_0

- Here, 95% CI corresponds to $\alpha = 0.05$ for a two-sided test. (100% - 95% = 5% and therefore, α=0.05)

Step 2: Check if $\mu_0 = 50$ is inside the CI

$$CI = (51.2, 59.8)$$

50 is not in the interval → reject H_0at 5% significance level.

This means the p-value < 0.05 since we rejected H_0 and if the p-value is LESS than 5%, it must also be less than 10%, so we also reject at the 10% level of significance

Step 3: Consider 10% significance level

- 95% CI corresponds to 5% significance level.
- If we increase α to 10%, the corresponding CI would be wider, so $\mu_0$is even more likely to be outside the CI.
- Therefore, we also reject H_0 at 10% significance level.

✅ Answer A. We reject H_0 at both 5% and 10% levels of significance

R21. The population variance of scores on a mathematics test is known to be $\sigma^2 = 64$. A random sample of 30 students from a local high school takes the test and obtains a sample standard deviation of $s = 10$.

Test, at the 5% level of significance, whether the variability of these students' test scores differs significantly from the population variance. Assume scores are normally distributed.

Step 1: State hypotheses

$H_0{:}\,\sigma^2 = 64$ vs. $H_a{:}\,\sigma^2 \neq 64$ (This is about variance and not a mean, so it is a Chi Square test)
Step 2: n=30, s=10, α00.05

Test statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$
$$n = 30, s^2 = 10^2 = 100, \sigma_0^2 = 64$$
$$\chi^2 = \frac{(30-1)(100)}{64} = \frac{29 \cdot 100}{64} = \frac{2900}{64} \approx 45.31$$

Step 3: Critical values (two-tailed, α = 0.05)

Degrees of freedom: $df = n - 1 = 29$

From chi-square tables:

$$\chi^2_{0.025,29} \approx 16.05, \chi^2_{0.975,29} \approx 45.72$$

Step 4: Compare test statistic to critical values

$$16.05 < 45.31 < 45.72$$

✅ Test statistic falls between the critical values, so do not reject H_0. It doesn't lie in either of the rejection regions.

Step 5: Conclusion

There is no significant evidence at the 5% level that the variability of the high school students' scores differs from the population variance.

Answer choices:

A. Reject H_0; variability is significantly different
B. Do not reject H_0; variability is not significantly different
C. Reject H_0; variability is not significantly different
D. Do not reject H_0; variability is significantly different

✅ Correct answer: B

R22.A normal population has an unknown standard deviation. A 95% confidence interval for the mean based on a sample of size $n = 12$ is $(8.21, 11.79)$, and the sample variance is 4.

What confidence level was actually used?

A. 90%
B. 95%
C. 99%
D. None of the above

Solution Step 1: Recall the CI formula for a normal population with unknown σ

$$\bar{x} \pm t_{\alpha/2,n-1} \cdot \frac{s}{\sqrt{n}}$$

Where: $s = \sqrt{\text{variance}} = \sqrt{4} = 2$

- $n = 12 \implies df = n - 1 = 11$
- $\bar{x} = \text{midpoint of CI} = \frac{8.21+11.79}{2} = 10$
- Half-width of CI:

$$E = \frac{width}{2} = \frac{11.79 - 8.21}{2} = 1.79$$

Step 2: Solve for $t_{\alpha/2}$

$$E = t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}$$

$$1.79 = t_{\frac{\alpha}{2}} \cdot \frac{2}{\sqrt{12}}$$

$$t_{\alpha/2} \approx 3.10$$

Step 3: Find the confidence level

- Degrees of freedom: $df = 11$
- Look up $t_{11,\alpha/2} \approx 3.10$ in t-tables:

This corresponds roughly to a 99% confidence level. (0.005 in each tail, or 99% in the centre)

✅ Answer A. 99%

R23.The Adventure Club wants to estimate how long teenagers take to complete the Riverside Trail. They want to construct a 95% confidence interval where the sample mean is within 8 minutes of the population mean. The standard deviation of the hiking time is 16 minutes.

What is the minimum number of teenagers that need to be surveyed?

Step 1: Recall the formula for sample size

For a confidence interval with known σ:

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

Where:

- $z_{\alpha/2}$= z-value for desired confidence level
- σ= population standard deviation
- E= desired margin of error

Step 2: Identify values

- Confidence level: 95% → $z_{\alpha/2} = 1.96$
- Standard deviation: $\sigma = 16$
- Margin of error: $E = 8$

Step 3: Plug into formula

$$n = \left(\frac{1.96 \cdot 16}{8}\right)^2$$
$$n = \left(\frac{31.36}{8}\right)^2$$
$$n = (3.92)^2 \approx 15.37$$

Step 4: Round up

Since we cannot survey a fraction of a person:

$$n = 16$$

✅ Answer **16 teenagers**

R24. A research company wants to calculate a 95% confidence interval for the proportion of people who like a new product, with a width of no more than 0.10. A previous 95% CI was calculated to be (0.42, 0.50).

What is the minimum sample size needed?

A. 97
B. 96
C. 100
D. 94
E. None of the above

Step 1: Identify the formula for sample size for a proportion

$$n = \mathrm{p}(1-\mathrm{p})\left(\frac{Z_{\frac{\alpha}{2}}}{E}\right)^2$$

Where:

- $Z_{\alpha/2}$= critical Z-value for the confidence level
- p= estimated proportion
- E= desired margin of error = half the width of the CI

Step 2: Find the values

- 95% CI → $Z_{\alpha/2} \approx 1.96$
- **Previous CI = (0.42, 0.50) We use 0.5 (closest endpoint to 0.5)**
- **Width = 0.10 → margin of error $\boldsymbol{E = \frac{width}{2} = \frac{0.10}{2} = 0.05}$**

Step 3: Plug into the formula

$$n = \mathrm{p}(1-\mathrm{p})\left(\frac{Z_{\frac{\alpha}{2}}}{E}\right)^2 = 0.5(1-0.5)\left(\frac{1.96}{0.05}\right)^2$$

$$n = \approx 384.16$$

✅ Step 4: Round up

- Minimum sample size = 385, so the answer is E).

R25. A study of high school students shows that they spend an average of 10 hours per week on social media, with a standard deviation of 2.5 hours. You want to test if this is true. You take a SRS of 36 students and find a sample mean of 11.2 hours.

a) What is the rejection region at the 5% level of significance for a two-tailed test?

- A. Reject H_0 if $Z_{\text{test}} > 1.96$ or $Z_{\text{test}} < -1.96$
- B. Reject H_0 if $Z_{\text{test}} > 1.645$ or $Z_{\text{test}} < -1.645$
- C. Reject H_0 if $Z_{\text{test}} > 2.575$ or $Z_{\text{test}} < -2.575$
- D. Reject H_0 if $Z_{\text{test}} > 1.28$ or $Z_{\text{test}} < -1.28$
- E. None of the above

b) What is the p-value of this test?

- A. 0.0456
- B. 0.0228
- C. 0.0114
- D. 0.1779
- E. None of the above

Solution

Step 1: Identify the hypotheses

$H_0: \mu = 10$ vs $H_a: \mu \neq 10$ (2-sided test)
Two-tailed test.

Step 2: Compute the Z-test statistic

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

- $\bar{X} = 11.2$
- $\mu = 10$
- $\sigma = 2.5$
- $n = 36$

$$Z = \frac{11.2 - 10}{\frac{2.5}{\sqrt{36}}}$$

$$Z \approx 2.88$$

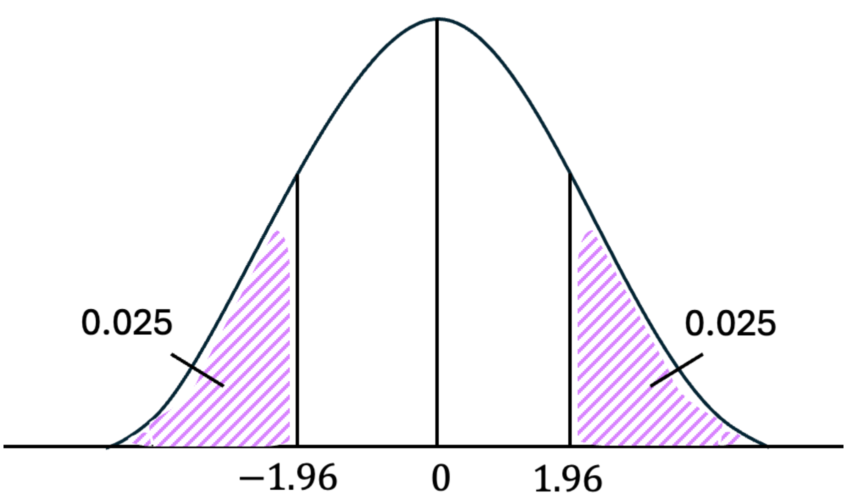
Step 3: Determine the rejection region (critical method)

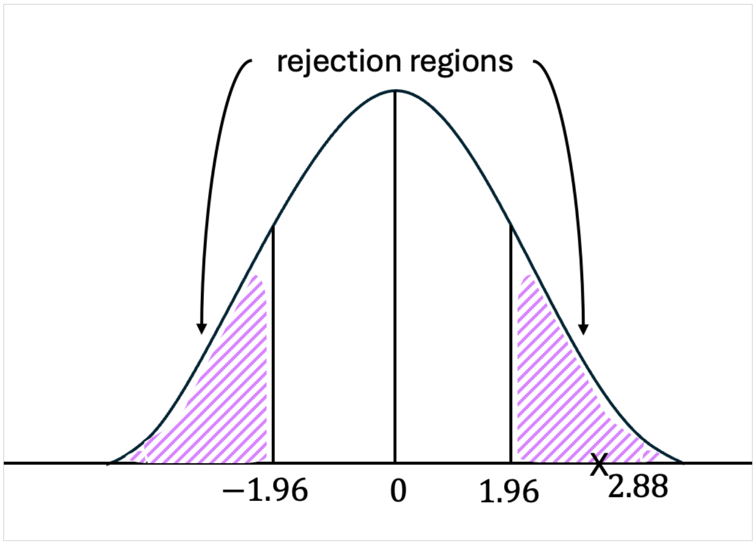
- 5% significance → $\alpha = 0.05$
- Two-tailed → $Z_{\alpha/2} = Z_{0.025} \approx 1.96$

$$\text{Reject } H_0 \text{ if } Z_{\text{test}} > 1.96 \text{ or } Z_{\text{test}} < -1.96$$

The rejection regions correspond to alpha/2 = 0.05=2 = 0.025 on each side

Look up area 0.025 in the body of the Z table and get -1.96. It is symmetrical, so the positive is the same number.





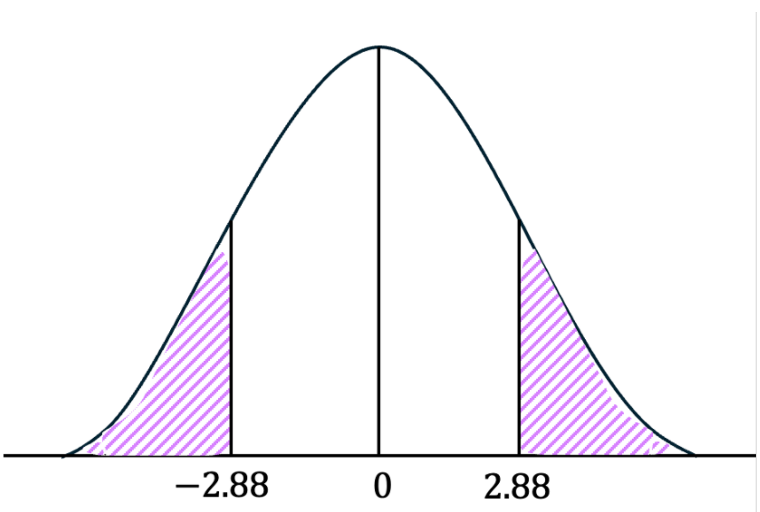
Calculated $Z = 2.88 > 1.96 \rightarrow$ reject H_0 i.e. 2.88 is in the rejection region on the right side

✅ Answer (a): A. Reject H_0 if $Z_{\text{test}} > 1.96$ or $Z_{\text{test}} < -1.96$

Step 4: Find the p-value

- Two-tailed test $\rightarrow p$-value $= 2P(Z > | 2.88 |)$
- i.e. since it is a 2-sided test, we plot BOTH Z= 1.88 and Z = − 1.88
- From standard normal table: $\Pr(Z < -1.88) = 0.0020$

p-value $= 2 \cdot 0.0020 = 0.0040$ $(2 - \textit{sided so we double the p} - \textit{value on one side})$



✅ Answer is E). None of the above

R26. You wish to test the hypothesis that the proportion of people who prefer online learning is different than 70%. In a sample of 150 people, 90 said they preferred online learning.

Which conclusion is correct?

A. We reject at the 5% level, but not at the 1% level
B. We cannot reject at both the 1% and 5% levels of significance
C. We cannot reject at the 5% or 10% level
D. We can reject at both the 1% and 5% level

Step 1: Set up the hypotheses: $H_0: p = 0.70$ vs $H_a: p \neq 0.70$ (2-sided test)
Two-tailed test for a proportion.

Step 2: Compute sample proportion

$$\hat{p} = \frac{\text{number preferring online}}{\text{sample size}} = \frac{90}{150} = 0.6$$

Step 3: Compute the test statistic

For a proportion: $Z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$

- $p_0 = 0.70$
- $n = 150$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.60 - 0.70}{\sqrt{\frac{0.70 \cdot 0.30}{150}}} = -2.67$$

Step 4: 2-sided test, so plot both Z= 2.67 and Z= - 2.67

p-value = 2(Pr(Z<-2.67)= 2(0.0038)=0.0076

✅ Answer p-value=0.0076< 1%, 5% and 10% levels, so reject H0 at all alpha levels

The answer is D).

Best of luck on the exam!