STAT 2035 Midterm 1 ACE Booklet Solutions (2024)

A. Variables

Example 1. $2^6 = 64 < 150$ $n = 150$

$2^7 = 128 < 150$

$2^8 = 256 > 150$

$\therefore k = 8$ $length = \frac{range}{k}$

$= \frac{100-36}{8}$

$= 8$

Example 2. Given the data below, complete the chart:

52.6, 68.4, 66.5, 75.0, 60.5, 78.8, 63.5, 48.9, 81.3
n=9
The data is in 1000's and represents salaries.

Step 1. Find k

2^3=8<n=9
2^4=16>9...so k=4

Step 2. Find the width of the classes

width= $\frac{range}{k} = \frac{81.3-48.9}{4} = 8.1 \ldots let's\ use\ 8.5\ or\ in\ 1000's\ 8500$

Step 3. Complete the chart

Boundaries	Midpoint	Frequency	Relative Frequency	Cumulative Frequency
[45000, 53500)	49,250	2	2/9=0.22	2
[53,500, 62,000)	57750	1	1/9=0.11	2+1=3
[62,000, 70,500)	66250	3	0.33	3+3=6
[70,500, 79,000)	74750	2	0.22	8
[79,000, 87,500)	83250	1	0.11	9

Example 3.

stem	leaf
3	2
4	4
5	
6	5 6 7
7	6 7
8	0 1 9

Common Statistical Graphs
Categorical= pie chart, bar chart, pareto chart
one quantitative variable= histogram, frequency polygon, ogive, stem and leaf plot
two quantitative variables=scatter plot

p.20

$$\frac{10+11}{2} = 10.5$$

306 to 402 = 96

$$\therefore \frac{f}{n} = \frac{96}{710} = 0.135\ (relative\ frequency)$$

(horizontal, vertical)= (10.5, 0.135)

Example 4.
A pareto graph is a bar graph where the bars are in decreasing order. It is for categorical data. The answer is D).

Example 5.
The answer is B) a pie chart. It is not quantitative data as they talk about each faculty.

Example 6.

A) 60/140=0.43 The answer is A).

B) 90/180=0.50 The answer is B).

Example 7.

a) Car=6/10=60%
Train= 2/10=20%
Plane=2/10=20%
b) (2+1)/6 = 3/6 = 50%
c) 12/16 = 0.75 or 75%

Example 8.
12/16 = 75%

A1. The answer is b).

A2. The answer is c).

A3. The answer is b).

A4. Conditional distribution of origin for students?

a) American=90/195=0.46
European=35/195=0.18*
Asian=70/195=0.36

b) $\frac{55}{95+25+55} = \frac{55}{175} = 0.31$

A5. The answer is d).

A6. The data is quantitative, so a bar graph or pie chart are not appropriate.

The answer is c).

A7. The answer is a).

Median=$\frac{n+1}{2} = \frac{21}{2} = 10.5$ ...the median is an average of the 10th and 11th numbers in order

The average of 36 and 37 = 36.5
A8. False. There is only one bar that is the highest.

A9. The categorical variables are present major, plans after graduation. All of the others are quantitative variables.

A10. The answer is d).

A11. The answer is c).

A12. The answer is b). Raw data has not been processed for use. It is also ratio data since 0 means 0 years old.

A13. 45/160=0.28

50+40+45+20= 155

midpoint = (5+10)/2 = 7.5

A14. The answer is a).

A15. The answer is d).

A16. The answer is c).

A17. The answer is a).

A18.
n=100

find smallest k so that $2^k>100$
$2^6=64$
$2^7=128>100$
k=7

range=36

length=range/k
=36/7 = 5.1...use 6 (don't round down)

A19.
University students were asked how likely they think it will be that they earn a 6-digit salary in the next 20 years.

Opinion	Female	Male
Almost no chance	200	150
Some chance, but not likely	400	300
A 50-50 chance	500	400
A good chance	400	700
Almost certain	100	300

a) How many individuals are described using this table?

1600+1850=3450

b) How many males are among those surveyed?

1850

c) Find the percent of females among the respondents.

$$\frac{1600}{3450} \times 100 = 46\%$$

d) Does part c) represent a marginal or conditional distribution? Why?

It represents the marginal distribution of sex.

e) What percent of females thought they had a good chance to earn 6-figures in the next twenty years?

$$\frac{400}{1600} \times 100 = 25\%$$

f) Does part e) represent a marginal or conditional distribution? Why?
The conditional distribution of chance to earn 6-figures among females.

A20.

a) How many employees are there in this company? 420

b) What percentage of employees are in management? $\frac{80}{420} \times 100 = 19\%$

c) What type of distribution does your answer to part b) represent?

The marginal distribution of employees.

d) What percentage of employees take a car?

$$\frac{82}{420} \times 100 = 19.5\%$$

e) What type of distribution does your answer to part d) represent?

The marginal distribution of mode of transportation.

f) What percentage of management take a train?

$$\frac{34}{80} \times 100 = 42.5\%$$

g) What type of distribution does your answer to f) represent?

Conditional distribution

A21. A) 35/90=38.8% b) 15/60=25%

B. Introduction to Statistics and Business Analytics

Example 1.

a) $interval\ -$ 0 doesn't mean absence of the variable
b) $ordinal$
c) $ratio$
d) $interval$
e) $ratio$
f) $ratio$
g) $ordinal$

Example 2.
interval b). has no meaningful zero...ratio does have a meaningful zero...quantitative is not unique to interval because both interval and ratio are quantitative.

Example 3.
The answer is A) because age, height and amount of loans are the only quantitative variables listed

Example 4.
The ordinal variables are education level and resort star ratings so the answer is B).

Example 5. The answer is C).

Example 6. The answer is A).

Example 7. The answer is C).

Example 8. The answer is A).

Example 9. The answer is E) since C) refers to Predictive Analytics and d) refers to Prescriptive Analytics.

Example 10. The answer is E).

B1.
a) ordinal
b) nominal
c) ordinal
d) ordinal
e) ratio
f) nominal
g) ordinal
h) interval
i) ratio

B2. Classify each of the following as ordinal, nominative, ratio or interval variables.

a) ratio

b) interval

c) ordinal

d) Nominal

e) ordinal

f) nominal

g) interval

**with a ratio interval, 0 means nothing
**temperature of 0 doesn't mean no temperature, but weight and height of 0 would mean nothing so they would both be ratio

h) ratio

i) ratio

B3. The answer is d). ordinal

B4. The answer is c).

B5. The answer is d).

B6. The answer is b).

B7. The answer is a) as it refers to Descriptive Analytics.

B8. The answer is e).

B9. The answer is c).

C. Measures of Centre

Example 1.
Draw a right skewed graph and the mean is pulled to the tail and the mean is greater than the median. The answer is a).

Example 2.
65, 66, 68, 68, 70 / 72, 75,77, 77, 77
a) Mode = 77
Median=$\frac{70+72}{2} = 71$ Mean= $\frac{65+66+\cdots+77}{10} = \frac{715}{10} = 71.5$

b) If we don't know the value of the 11th student's mark we cannot calculate the median of the marks
c) if the mean is 72, let the missing students mark be x and solve the equation below

$$\frac{75+72+77+70+77+68+65+77+66+68+x}{11} = 72$$

We get 72(11)=715+x when we add up the numbers on top
Solving for x, x=77

Example 3.

Class	Midpoint	Frequency
(10, 15)	12.5	5
(15, 20)	17.5	20
(20, 25)	22.5	35
(25, 30)	27.5	30
(30, 35)	32.5	26

$$\bar{x} = \frac{5(12.5)+20(17.5)+35(22.5)+30(27.5)+26(32.5)}{116}$$
$$= \frac{2870}{116}$$
$$= 24.74$$

Example 4.
$The\ answer\ is\ B).\quad mean\ pulled\ to\ tail$

Example 5
The answer is D) because we do not talk about the skew of categorical data.

C1.

Mode- occurs most often...Therefore, 67 and 78 (bi-modal)

Median- Write numbers in ascending order and take the middle # which is 67.
34, 44, 50, 56, 66, 67, 67, 78, 78, 88, 98

Mean=$\frac{34+44+\cdots+98}{11}$ =66

C2.
Originally, the numbers are 60, 70, 80

Median= 70 (middle # when written in ascending order)

Mean=$\frac{60+70+80}{3} = 70$

After adding a mark of 75... the numbers are 60,70,75,80

Median=$\frac{70+75}{2} = 72.5$

Mean=$\frac{60+70+75+80}{4} = 71.25$

The answer is c).

C3.

$$\frac{80 + 75 + 95 + x}{4} = 80.5$$

80+75+95+x = 322

x = 322 - 95 - 75 - 80
x=72

Therefore, the mark on the fourth test was 72.

C4.

The numbers represented by the stemplot are 53, 57, 62, 63, 65, 68, 71, 74, 83, 92
Median=$\frac{65+68}{2} = 66.5$
Mean=$\frac{53+57+\cdots+92}{10} = 68.8$

C5. The answer is (c). Think of the Normal distribution as an example.

C6. Both a stemplot and boxplot reveal the shape as if you flip them sideways, you can tell the skew from both of them. Stemplots are NOT better for large data sets, since they display every number across the page. So, the answer is III. since a stemplot does show every number. the answer is C.

C7. This has a tail to the right, so it is positively or right skewed. The answer is C.

C8. Since there are 119 students, the median occurs at (n+1)/2=(119+1)/2=60th data... (50% of the data above and below since data is in percent)

The first four bars are 1+2+15+24=42, so the median occurs in the tallest bar, the next bar over to the right.

Therefore, the median is approximately 10 pounds. The answer is B.

C9. 4, 5, 5, 5, 6, 6, 7, 8, 11, 13

(a) The median is the middle # = average of 6 and 6 = 6...answer is B

(b) The mode is 5, since it occurs the most often...answer is A

(c) To find the mean, add up all data and divide by 10 numbers...mean is 7...answer is C.

C10.
?=200 - 50 - 80 - 30 = 40

$$\text{Mean}=\frac{5(50)+15(80)+25(40)+35(30)}{200}=17.5$$

D. Measures of Variation

Example 1.

a) 55, 66, 67, 78, 79, 80, 85, 86, 87, 90, 92

$Q1 = \frac{25}{100} \times 11 = 2.75 \quad \therefore 3rd\ number \quad \therefore Q1 = 67$

b) $30th\ P = \frac{30}{100} \times 11 = 3.3 \quad \therefore 4th\ number \quad \therefore = 78$

c) $percentile = \frac{4}{11} \times 100 = 36th$

Example 2.

$\bar{x} = \frac{195}{5} = 39$

$s^2 = \frac{(35-39)^2+(25-39)^2+(75-39)^2+(15-39)^2+(45-39)^2}{4}$

$s^2 = \frac{16+196+1296+576+36}{4}$

$s^2 = 530$

$s = \sqrt{530}$

$= 23.02$

$$s^2 = \frac{9725 - \frac{195^2}{5}}{4} = 530$$

Example 3.

$CV = \frac{\sigma}{\mu} \times 100$

$= \frac{4}{40} \times 100 = 10$

Example 4.

$CV = \frac{s}{\bar{x}} \times 100$

$25 = \frac{4}{\bar{x}} \times 100$

$25\bar{x} = 400$

$\bar{x} = 16$

Example 5.

1 Canada $\frac{10}{75} = 0.133$

2 Britain $\frac{6}{30} = 0.2$

3 Switzerland $\frac{14}{85} = 0.165$

$1 < 3 < 2$

Example 6.

$n = 10 \qquad median = \frac{n+1}{2} = \frac{11}{2} = 5.5$

Therefore, 180 to 190 has the 5th and 6th numbers in order

$Q1 = \frac{25}{100} \times 10 = 2.5 \quad \therefore 3rd\ number$

Therefore, 180 to 190.

I. true
II. not true
III. true
IV. not true – median for skewed data
V. true $\frac{80}{100} \times 10 = 8 \quad \therefore average\ 8th\ and\ 9th$

The answer is B.

Example 7.

Female $range = max - min$
$= 24 - 14$
$= 10$

Male $range = 30 - 15$
$= 15$

A. false
B. false
C. false, it is 23.
D. false $IQR\ male = Q3 - Q1 = 26 - 19 = 7$
E. true $IQR\ \ female = 20 - 17 = 3$

The answer is E.

Example 8.

$Q1 = 17 \quad Q3 = 20 \quad IQR = 3$

Outliers below $Q1 - 1.5(IQR)$
$= 17 - 1.5(3) = 12.5$

Above $Q3 + 1.5(IQR)$
$= 20 + 1.5(3) = 24.5$

$\therefore 27, 29, 31, 33\ \ are\ all\ outliers$

Example 9.

x, -1, 0.5, 0.8, 1.7, 2.2, 3.1, 4.8, 5.5, 6.2, 7.4

$Q1 = \frac{25}{100} \times 11 = 2.75 \quad \therefore 3rd\ \# = 0.5$

$Q3 = \frac{75}{100} \times 11 = 8.25 \quad \therefore 9th\ \# = 5.5$

$IQR = Q3 - Q1 = 5.5 - 0.5 = 5$

Below $Q1 - 1.5(IQR)$
$= 0.5 - 1.5(5) = -7\ below$

$\therefore The\ answer\ is\ a)\ only$

Example 10.
The set with the smallest deviation would be the set of data that is the least spread out. You could calculate all the standard deviations and they are 5.77, 1, 6.45, 2.58 but you should be able to just tell that B) would be the smallest deviation because the data is so close together.

Example 11.
a) n=17 data
Q1=0.25x17=4.25, so we take the 5th data in order from the smallest part of the stemplot and Q1=63
Q3=0.75x17=12.75, so we take the 13th data in order, and Q3=76
IQR=Q3-Q1=76-63=13

b) The 68th percentile is 0.68x17=11.56, so we round up and take the 12th number in order and it is 75.

Example 12.
Write the numbers in order, including the 12th mark of 72

65, 66, 68, 68, 70, 72, 72, 75, 77, 77, 77, 80
Q1=0.25x12=3 so we average the 3rd and 4th numbers

$$Q1 = \frac{68 + 68}{2} = 68$$

Q3=0.75x12=9, so we average the 9th and 10th numbers in order

$$Q3 = \frac{77 + 77}{2} = 77$$

IQR=Q3-Q1=77-68=9

D1. (ii) The distribution is skewed to the right, so the mean is larger than the median.

(b) $\frac{16}{20} \times 100\% = 80\%$. The answer is (iv).

(The first three bars= 6+5+5 = 16)

(c) The correct stemplot you choose earlier for this data was...

2|011355
3|01467
4|12479
5|56
6|
7|
8|0
9|
10|5

median= (n+1)/2 = 21/2 = 10.5...average of 10th and 11th data

The upper half of the data set that is above the median consists of the data points

37, 41, 42, 44, 47, 49, 55, 56, 80, 105.

Q3=75/100x20=15...average of 15th and 16th data

The third quartile is the median of the above list: $Q_3 = \frac{47+49}{2} = 48$.

(d) **IQR=23**

There are no outliers in the lower end (the distribution does not have a long left tail).

$Q_3 + 1.5\,IQR = 48 + 1.5 \times 23 = 82.5$. Any number above 82.5 is an outlier.

Since $105 > 82.5$, the data point 105 is an outlier, the only outlier in the data set.

(e) Both the median and IQR are insensitive to outliers, so in the presence of outliers (105), we should report these two summary statistics. The answer is (iii).

f) relative frequency=6/20=0.3

D2.
23, 46, 49, 51, 64, 64,** 67,** 81, 88, 89, 97, 103, 121

If we look, we can see that the data is in order. There are 13 data points, so the median is the 7th data which is 67.

Q_1 =25/100 x n = 0.25 x 13 =3.25...so round up and use the 4th number.
Q1=51
median=67
Q3= 75/100xn=0.75x13=9.75...so round up and use the 10th number
Q3=89
$Range = 121 - 23 = 98$
IQR=Q3-Q1=89-51=38
We can use the formulas above to calculate the mean and sample standard deviation:

$$\bar{x} = \frac{23 + 46 + \cdots 97 + 103 + 121}{13} = 72.5$$

$$s = \sqrt{\frac{(23-72.5)^2 + (46-72.5)^2 + \cdots (97-72.5)^2 + (103-72.5)^2 + (121-72.5)^2}{12}}$$

$$= \sqrt{\frac{8806.4172}{12}} = 27.09$$

We now check for outliers:

$$Q_3 + 1.5\ IQR = 89 + 1.5(38) = 146$$
$$Q_1 - 1.5\ IQR = 51 - 1.5(38) = -6$$

Since no data point is above 146 or below -6, there are no outliers in the data set.

D3. 37 reactions.

15	5
16	
17	
18	
19	0 1 4
20	1 2 3 5 5 9
21	1 1 4 5 6 8 9 9
22	0 2 4 5 6 6 7 8 9 9
23	0 0 1 3 7
24	2 4 4 5

(a)

Unimodal (one peak).
Asymmetric – skewed to the left.
Has a suspected outlier (155g).

(b)

With the wrong data: $\bar{x} = \frac{155+190+191+\cdots 244+245}{37} = 8070/37 = 218.1$

With the correct data: $\bar{x} = \frac{195+190+191+\cdots 244+245}{37}$

$= \frac{8070+40}{37} = 8110/37 = 219.2$

(ii) Summary Statistics

median	III
IQR	III
standard deviation	I
75^{th} percentile	III

D4. Write the numbers in ascending order: n=9
35, 50, 55, 65, 65, 70, 80, 80, 95

Mode= 65, 80 (bi-modal)

Median=middle # = 5th number = 65

Mean=$\frac{35+50+\cdots+95}{9} = 66.1$ (use 66 for simplicity)

Range= max - min= 95 - 35 = 60

Make a chart to find the standard deviation:

$x_i - \bar{x}$	$(x_i - \bar{x})^2$
35-66	961
50-66	256
55-66	121
65-66	1
65-66	1
70-66	16
80-66	196
80-66	196
95-66	841

$$s = \sqrt{\sum \frac{(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{2589}{8}} = 17.99$$

D5. The answer is c).

D6. The answer is b).

D7. The answer is a).

D8. 29, 56, 59, 62, <u>66</u>, 67, 78, 81, 89

IQR= Q_3 - Q_1
First, we need the median and then the 1st and 3rd quartiles

Median=66

Q1=25th percentile=25/100x9=2.25..round up and use the 3rd number=59

Q3=75 percentile=75/100x9=6.75...round up and use the 7th number=78

IQR=Q3-Q1=78-59=19

<u>To find outliers</u>

Q3 + 1.5 IQR= 78 + 1.5(19) = 106.5

Q1 - 1.5IQR= 59 - 1.5(19)= 30.5

An outlier is any number above 106.5 or below 30.5.

Therefore, there is one outlier...the number "29".

D9. 21, 23, 26, 27, 28, 32, 34, 34, 37, 38, 40, 41

$$\bar{x} = \frac{21 + 23 + \cdots + 41}{12} = 31.75$$

Make a chart to find the standard deviation.

$x - \bar{x}$	$(x - \bar{x})^2$
21-31.75	115.56
23-31.75	76.56
26-31.75	33.06
27-31.75	22.56
28-31.75	14.06
32-31.75	0.06
34-31.75	5.06
34-31.75	5.06
37-31.75	27.56
38-31.75	39.06
40-31.75	68.06
41-31.75	85.56

$$s = \sqrt{\frac{478.16}{11}} = 6.6$$

The variance is $s^2 = 43.5$

Outliers?

Q1=0.25(12)=3...average of 3rd and 4th data= (26+27)/2 = 26.5

Q3=0.75(12)=9...average of 9th and 10th=(37+38)/2 = 37.5

IQR= 37.5 - 26.5 = 11

Q1- 1.5(IQR)= 26.5 - 1.5(11) = 10 below

Q3+ 1.5(IQR) = 37.5 + 1.5(11) = 54 above

There are no numbers below 10 or above 54, so there are no outliers!

D10. $CV = \frac{s}{\bar{x}} \times 100 = \frac{3.5}{69.5} \times 100 = 5.04\%$

D11. 0,0,100,100,200,250,250,300,400,400,500,500,750,900,1000

30th percentile=$\frac{30}{100} \times 15 = 4.5 \ldots round\ up\ to\ 5th\ piece\ of\ data \ldots 200$
So, 200 is the 30th percentile

D12.

A). Since the data is grouped, to find the mean, we use the midpoint of each interval.

Mean=$\frac{57(5)+62(5)+67(10)+72(10)+77(5)}{35} = 67.7$
B) The variance is $\frac{5(57-67.7)^2+5(62-67.7)^2+10(67-67.7)^2+10(72-67.7)^2+5(77-67.7)^2}{34} = 39.9$

C) Take the square root of the variance, to find the standard deviation
standard deviation=$\sqrt{39.9}$=6.32

D13.
A). Yellow board...median is closest to 45...answer is (a).

B). Green board Q1=25 and Q3=33 and IQR=33-25=8
answer is (b).

C). The shape of the beetles on green boards is left skewed or negatively skewed. The answer is (a). (long tail to the left, it you flip the Box plot on its side)

D14. This graph is skewed to the right, so we need to use a measure that is resistant. The mean is not a resistant measure of the centre. The standard deviation is not resistant and since the question asks about centre, it couldn't be standard deviation anyway as it measures spread. So, we would use the median, since it is a measure of centre and it IS resistant.

The answer is (b).

D15.
a) median occurs at (n+1)/2 = (100+1)/2 = 101/2=50.5...average of the 50th and 51st numbers

5+18= 23...not in the second bar

5+18+42=65...too far, so the median is in the third bar...
between 66 and 69

b) 0.25(100)=25...average of 25th and 26th data
first quartile= 25% lies below it...so 25 people below it...so it would be in the 66 to 69 range as well since 5+18=23 isn't quite 25%

c) 0.75(100)=75...average of 75th and 76th data
the third quartile means 75% lie below it
5+18+42+27=92...so it is somewhere after the third bar since it added up to 65 which wasn't large enough, so the third quartile is in between 69 and 72

D16.
At the start, $\bar{x} = 80, n = 4, s = 3$

Add new number x_5=80 and now, n=5

From the original data, $\bar{x} = \frac{\sum x}{n}$, so $\sum x = n\bar{x} = 4(80) = 320$

New mean= $\frac{320+80}{5} = \frac{400}{5} = 80$

Old s^2=9

$$s^2 = \frac{\sum(x-\bar{x})^2}{n-1}$$

9=$\frac{\sum(x-\bar{x})^2}{3}$

$$\sum(x-\bar{x})^2 = 27$$

New standard deviation=$\sqrt{\frac{27+(80-80)^2}{5-1}} = \sqrt{6.75} = 2.6$

D17. min=35 Q1=68, Median=77, Q3=83 and Max=97

The number of scores between 77 and 83 is the number of scores from the median to Q3 which is 25% of the scores, so 0.25x196=49 scores. The answer is (c).

D18. 2, 12, y,y,y,15, 18, 18, 19

Mean is 13.6666

There are 9 numbers

We can find the mean...

$$\frac{2+12+y+y+y+15+18+18+19}{9}=13.6666$$

Cross-multiply and solving for y...we get:

14+3y+70=123
3y=39
y=13

Now, the numbers are 2,12, 13, 13, 13, 15, 18, 18, 19

Median=(n+1)/2= (9+1)/2= 5th data
...median is 13

The median is 13. I is false

The mode is 13. II is true

Q1=0.25x9=2.25...3rd # = Q1=13
Q3=0.75x9=6.75...8th # = Q3=18
IQR=18-13=5

An outlier would be below Q1 -1.5(IQR)=
below 13 -1.5(5)=5.5...so "2" is an outlier
III is true

The answer is (b).

D19. The first quartile occurs at 0.25 x 50=12.5, so at the 13th observation, so the answer is a) 0 to 10.

E. Weighted Means

Example 1.
Unweighted mean= (10+20+10+5+15)/5=12%

Weighted mean

$$= \frac{10(10{,}000) + 20(20{,}000) + 10(50{,}000) + 5(30{,}000) + 15(100{,}000)}{10{,}000 + 20{,}000 + 50{,}000 + 30{,}000 + 100{,}000}$$

=2650000/210000
=12.62%

Example 2.

$$\bar{x} = \frac{5(12.5) + \cdots + 26(32.5)}{116} = 24.74$$

Class	M_i	f_i	f_iM_i	$M_i - \bar{x}$	$(M_i - \bar{x})^2$	$f_i(M_i - \bar{x})^2$	
[10,15)	12.5	5	62.5	-12.24	149.8176	749.088	
[15,20)	17.5	20	350	-7.24	52.4176	1048.352	
[20,25)	22.5	35	782.5	-2.24	5.0176	175.616	
[25,30)	27.5	30	825	2.76	7.6176	228.528	
[30,35)	32.5	26	845	7.76	60.2176	1565.6571	

$n = 116$ $sum = 3767.24$

$$s^2 = \frac{\sum f_i(M_i - \bar{x})^2}{n-1} = \frac{3767.24}{116-1} = 32.76 \ variance$$

$$standard\ deviation \quad s = \sqrt{32.76} = 5.72$$

Example 3.

$l = 20 \quad w = 5 \quad f = 35 \quad F = 25 \quad n = 116$

$$median = \frac{n+1}{2} = \frac{116+1}{2} = 58.5$$

$$5 + 20 = 35 < 58.5$$

$$5 + 20 + 35 = 60 > 58.5 \qquad \therefore median\ is\ in\ [20, 25)$$

$$Median = l + \left(\frac{\frac{n}{2} - F}{f}\right) \times w \qquad median = 20 + \frac{\left\lfloor \frac{116}{2} - 25 \right\rfloor}{35} \times 5$$

$$= 20 + 4.714$$

$$= 24.714$$

Example 4.

$\mu = 24.74137 \qquad M = 24.714$

$$S_k = \frac{3(\mu - M)}{\sigma}$$

$$= \frac{3(24.74137 - 24.714)}{5.72} \qquad = 0.014 > 0 \quad \therefore right\ skewed$$

E1. Given the data below, calculate the weighted mean and the sample variance.

a)

Marks	Frequency	M_i	f_iM_i
60-64	5	62	310
65-69	10	67	670
70-74	5	72	360
75-79	20	77	1540
80-84	15	82	1230
85-89	30	87	2610
	n=85		Total 6720

$\bar{x} = \frac{6720}{85}$=79

f_i	M_i	$(M_i-\bar{x})$	$(M_i-\bar{x})^2$	$f_i(M_i-\bar{x})^2$
5	62	-17	289	1445
10	67	-12	144	1440
5	72	-7	49	245
20	77	-2	4	80
15	82	3	9	135
30	87	8	64	1920
n=85				Total 5265

$s^2 = \frac{\sum f_i(M_i-\bar{x})^2}{n-1} = \frac{5265}{84} = 62.7$

Median=$\frac{n+1}{2} = \frac{85+1}{2} = 43$...average of 43rd and 44th

5+10+5+20= 40 so median is not in the 1st to 4th classes...5+10+5+20+15>43 so the median is in the class 80 to 84 (class 5)

$l = lower\ endpoint\ of\ class\ containing\ median = 80$

$w = width\ of\ class\ containing\ median$=4

$f = frequency\ of\ class\ containing\ the\ median$=15

$F =$
$cumulative\ frequency\ of\ classes\ preceding\ the\ class\ containing\ the\ median$=5+10+5+20=40

$n = sample\ size = 85$

$Median = l + \left(\frac{\frac{n}{2}-F}{f}\right) \times w = 80 + \left(\frac{42.5-40}{15}\right) \times 4 = 80.666$

b)

Mass	Frequency	M_i	f_iM_i
1-3	20	2	40
4-6	30	5	150
7-9	10	8	80
10-12	15	11	165
13-15	25	14	350
	n=100		Total 785

$$\bar{x} = \frac{785}{100} = 7.85$$

f_i	M_i	$(M_i-\bar{x})$	$(M_i-\bar{x})^2$	$f_i(M_i-\bar{x})^2$
20	2	-5.85	34.2	684
30	5	-2.85	8.12	243.6
10	8	0.15	0.0225	0.225
15	11	3.15	9.92	148.8
25	14	6.15	37.82	945.5
n=100				Total 2022.1

$$s^2 = \frac{\sum f_i(M_i-\bar{x})^2}{n-1} = \frac{2022.1}{99} = 20.4$$

Median=$\frac{n+1}{2} = \frac{100+1}{2} = 50.5$...average of 50th and 51st

20+30 = 50 so median is not in second class...20+30+10>50.5 so the median is in the class 7 to 9

$l = lower\ endpoint\ of\ class\ containing\ median =$7

$w = width\ of\ class\ containing\ median$=2

$f = frequency\ of\ class\ containing\ the\ median$=10

$F =$
$cumulative\ frequency\ of\ classes\ preceding\ the\ class\ containing\ the\ median$=20+30=50

$n = sample\ size = 100$

$$Median = l + \left(\frac{\frac{n}{2}-F}{f}\right) \times w = 7 + \left(\frac{50-50}{10}\right) \times 2 = 7$$

E2. weighted average $=\frac{0.8(30)+0.2(40)}{0.8+0.2}=32$

E3.
weighted mean$=\frac{12(1.26)+13(1.2)+31(1.18)+9(1.32)+29(1.12)+13(1.25)}{12+13+31+9+29+13}=\$1.20$

E4.
You can just do this question with percentages, or assume you invest some amount of money and divide it according to the percentages given to you...eg. $10 000

$2500 in A, 5000 in B and $2500 in C

weighted mean$=\frac{6(2500)+6(5000)+2(2500)}{2500+5000+2500}=5\%$

E5. Given the data below, calculate the median.

Marks	Frequency
60-64	5
65-69	10
70-74	5
75-79	20
80-84	***15***
85-89	30

Median$=\frac{n+1}{2}=\frac{85+1}{2}=43$
5+10+5+20=40...so the next class has the median in it...class 5 (80-84) has the median in it

$l=lower\ endpoint\ of\ class\ containing\ median$=80

$w=width\ of\ class\ containing\ median$=4

$f=frequency\ of\ class\ containing\ the\ median$=15

$F=cumulative\ frequency\ of\ classes\ preceding\ the\ class\ containing\ the\ median$=5+10+5+20=40

$n=sample\ size$=85

$$Median=l+\left(\frac{\frac{n}{2}-F}{f}\right)\times w=80+\left(\frac{42.5-40}{15}\right)\times 4=80.7$$

b) Given the data below, calculate the median.

Mass	Frequency
1-3	20
4-6	30
7-9	***10***
10-12	15
13-15	25

Median=$\frac{n+1}{2}=\frac{100+1}{2}=50.5$
20+30=50...so the next interval contains the median= 3rd class (7-9)

$l = lower\ endpoint\ of\ class\ containing\ median$=7

$w = width\ of\ class\ containing\ median$=2

$f = frequency\ of\ class\ containing\ the\ median = 10$

$F = cumulative\ frequency\ of\ classes\ preceding\ the\ class\ containing\ the\ median = 20 + 30 = 50$

$n = sample\ size$=100

$Median = l + \left(\frac{\frac{n}{2}-F}{f}\right) \times w$=$7 + \left(\frac{50-50}{10}\right) \times 2 = 7$

E6. Add up all freq and we get n= 62

Frequency	Midpoint	f_iM_i
4	31	124
12	34	408
30	37	1110
6	40	240
10	43	430
Total 62		Total 2312

$\overline{x} = \frac{2312}{62} = 37.29$...use 37 for simpler calculations

f_i	M_i	$(M_i-\overline{x})$	$(M_i-\overline{x})^2$	$f_i(M_i-\overline{x})^2$
4	31	-6	36	144
12	34	-3	9	108
30	37	0	0	0
6	40	3	9	54
10	43	6	36	360
Total 62				Total 666

$s^2 = \frac{\sum f_i(M_i-\overline{x})^2}{n-1} = \frac{666}{61} = 10.9$

Satisfaction Rating	Frequency
30-32	4
33-35	12
36-38	***30***
39-41	6
42-44	10

Median=$\frac{n+1}{2} = \frac{62+1}{2} = 31.5$...average of 31st and 32nd

4+12=16 so median is not in second class...4+12+30>31.5 so the median is in the class 36 to 38 (class 3)

$l = lower\ endpoint\ of\ class\ containing\ median =$36

$w = width\ of\ class\ containing\ median$=2

$f = frequency\ of\ class\ containing\ the\ median$=30

$F =$
$cumulative\ frequency\ of\ classes\ preceding\ the\ class\ containing\ the\ median$=4+12=16

$n = sample\ size = 62$

$$Median = l + \left(\frac{\frac{n}{2} - F}{f}\right) \times w = 36 + \left(\frac{31 - 16}{30}\right) \times 2 = 37$$

Satisfaction Rating	Frequency
30-32	4
33-35	12
36-38	30
39-41	6
42-44	10

Coefficient of Skewness

The coefficient of skewness is a measure of how skewed a distribution is can be calculated as follows:

$\mu = mean$=37.29
$M = median$=37
$\sigma = \sqrt{10.9}$ (from example 2)

$S_k = \frac{3(\mu - M)}{\sigma} = \frac{3(37.29-37)}{\sqrt{10.9}} = 0.26$...therefore, it is positively skewed

E7. Given the following data, find the overall average?

Class	No. of Students	Average
1	120	78.2
2	100	67
3	95	77.1
4	125	75.2
5	88	77.5

Add up all of the students and we get n=528

$$\overline{x} = \frac{120(78.2) + 100(67) + 95(77.1) + 125(75.2) + 88(77.5)2312}{120 + 100 + 95 + 125 + 88} = \frac{39628.5}{528} = 75.1$$

F. Empirical Rule

Example 1.

$right = 50\% - 34\% = 16\%$

$left = 50\% - \frac{95}{2} = 2.5\%$

$total = 18.5\%$

Example 2.

$left = \frac{99.7}{2} = 49.85$ $right = \frac{95}{2} = 47.5$

$total = 97.35\%$

Example 3.

a) $Z1 = \frac{x-\mu}{\sigma} = \frac{60-67}{10} = -0.70$

b) $Z2 = \frac{x-\mu}{\sigma} = \frac{80-75}{5} = 1$

c) $Z3 = \frac{x-\mu}{\sigma} = \frac{75-80}{3} = -1.67$

Z3, Z2, Z1 is largest to smallest of relative standings (largest standard deviation away from mean to smallest)

F1. The middle 68% of the data is from $\mu - \sigma \; to \; \mu + \sigma$, ie. one standard deviation on either side of the mean

The mean is 72, so $\mu - \sigma = 72 - 10 = 62$
$\mu + \sigma = 72 + 10 = 82$

So, 68% of the marks lie between 62 and 82 and the lowest mark needed to get a C would be a 62.

F2. $\mu = 125 \text{ } and \text{ } \sigma = 10$

$\mu - 2\sigma = 125 - 2(10) = 125 - 20 = 105$
and

$\mu + 2\sigma = 125 + 2(10) = 145$

We know that from two standard deviations below the mean to two above the mean we have 95% of the data. So, from 105 to 145 makes up 95% of the data.

So, the percentage below 105 or above 145 is 100 - 95% = 5%.

F3. Mean =20 and standard dev=5

Find % between 15 and 40

See diagram to the right= 95/2 + 99.7/2 = 97.35%

F4.

Mean =8 and standard dev=2

Find % above 12 or below 6

On the right, to find above 12, we see that the unshaded portion is 95/2=47.5 so the shading above 12 would be 50% - 47.5%=2.5%

On the left of the mean from 6 to 8 would be 68/2%= 34%, so the shaded area we want would be 50% - 34% = 16%

The total shading is then 2.5% + 16%=18.5%

G. Chebyshev's Theorem

Example 1.

$\mu = \frac{30+70}{2} = 50 \qquad \sigma = 2$

$k = \frac{d}{\sigma} = \frac{20}{2} = 10$

$\therefore \% = 100(1 - \frac{1}{k^2})$

$= 100(1 - \frac{1}{10^2})$

$= 99\%$

Example 2.

$n = 200\ scores \qquad N = n(1 - \frac{1}{k^2})$

$\begin{bmatrix} 40 & 80 \\ a & b \end{bmatrix} \qquad \mu = \frac{40+80}{2} = 60(given)$

$\sigma^2 = 100 \qquad \sigma = 10$

$k = \frac{d}{\sigma} = \frac{20}{10} = 2$

$N = 200\left(1 - \frac{1}{2^2}\right) = 200(0.75) = 150\ scores$

Example 3.

a) $n = 250 \qquad N = n(1 - \frac{1}{k^2})$

$0.75 = 1 - \frac{1}{k^2}$

$0.75 - 1 = \frac{-1}{k^2}$

$-0.25 = \frac{-1}{k^2}$

$k^2 = 4$

$k = \pm 2$

$k = \frac{d}{\sigma}$ d=92-80=12

$\mu = 80 \qquad 75\%\ lie\ between\ 68\ \&\ 92 \quad [68, 92]$

$\therefore 2 = \frac{12}{\sigma} \qquad \therefore 2\sigma = 12$

$\sigma = 6$

b) $[67, 93]$

$\mu = 80 \qquad a = 67 \quad b = 93$

$k = \frac{d}{\sigma} = \frac{13}{6} = 2.1\overline{6}$

$N = n\left(1 - \frac{1}{k^2}\right) = 250\left(1 - \frac{1}{2.1\overline{6}^2}\right) = 196.7\ or\ 197\ scores$

G1. Chebyshev's

$$\begin{matrix} a & b \\ [22, & 58] \end{matrix}$$

a=22 and b=58

$$\mu = \frac{22+58}{2} = 40 \quad \sigma = 10$$

$$\mu - k\sigma = a$$

$$40 - 10k = 22$$

$$-10k = -18 \quad k = 1.8$$

$$* \, or \; use \; distance \; from \; \mu \; is \; d \; and \; k = \frac{d}{\sigma} = \frac{18}{10} = 1.8$$

$$N = n\left(1 - \frac{1}{k^2}\right) = 250\left(1 - \frac{1}{1.8^2}\right) = 69.1\%$$

=0.691 x 250 = 173 scores

G2.
a=10
b=60

$\mu = 35$ (centre of interval)
$\sigma = 4$
d=60-35=25
$\mu - k\sigma = a$
35 - 4k=10
4k=25
k=25/4=6.25
or use $k = \frac{d}{\sigma} = \frac{distance\ from\ \text{a}\ or\ "b"}{\sigma} = \frac{25}{4} = 6.25$
% of population is $100(1 - \frac{1}{k^2}) = 100\left(1 - \frac{1}{6.25^2}\right) = 97.44\%$

G3. =10
b=30

$\mu = 20$ (centre of interval)
$\sigma = 4$
$\mu - k\sigma = a$
20 - 4k=10
4k=10
k=2.5
or do $k = \frac{d}{\sigma} = \frac{distance\ from\ \text{a}\ or\ "b"}{4} = \frac{10}{4} = 2.5$
% of population is $100(1 - \frac{1}{k^2}) = 100\left(1 - \frac{1}{2.5^2}\right) = 84\%$

G4. Chebyshev's

$$n = 200 \quad \sigma^2 = 16 \quad \therefore \sigma_x = 4 \quad \mu = 95$$

$$N = n \times (1 - \frac{1}{k^2})$$

$$75 = 200(1 - \tfrac{1}{k^2})$$

$$\tfrac{75}{200} = 1 - \tfrac{1}{k^2}$$

$$0.375 - 1 = -\tfrac{1}{k^2}$$

$$-0.625 = -\tfrac{1}{k^2}$$

$$0.625k^2 = 1 \qquad k = \sqrt{1.6} = 1.26$$

$$a = \mu - k\sigma = 95 - 1.26(4) = 89.96$$

$$b = \mu + k\sigma = 95 + 1.26(4) = 100.04$$

$\therefore between\ 90\ and\ 100$

$\therefore between\ 90\ and\ 100$

G5.

[190, 310] $\mu = 250$

$\% = 100\left(1 - \frac{1}{k^2}\right)$

$90 = 100(1 - \frac{1}{k^2})$

$0.9 = 1 - \frac{1}{k^2}$

$0.1 = \frac{1}{k^2}$

$0.1k^2 = 1$

$k^2 = 10$

k=3.16

Find the standard deviation $\sigma\ using\ k = d/\sigma$

$3.16 = \frac{60}{\sigma}$ and $\sigma = \frac{60}{3.16} = 18.99$

Now, the new interval given is [150, 350] same mean=250

$k = \frac{d}{\sigma} = \frac{100}{18.99} = 5.27$

$\% = 100\left(1 - \frac{1}{5.27^2}\right) = 96.4\%$

H. Basic Properties of Probability

p.118

Example 1.

Crazy Question:

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A3' \cap B1') = 1 - \Pr(A3 \cup B1) \leftarrow union$

$= 1 - [\Pr(A3) + \Pr(B1) - \Pr(A3 \cap B1)]$

$= 1 - [0.46 + 0.27 - 0.11]$

$= \boxed{0.38}$

p.111

Example 2.

a) $\Pr(F \cup B) = \Pr(F) + \Pr(B) - \Pr(F \cap B)$

$= 0.65 + 0.39 - 0.25$

$= 0.79$

b) $\Pr(F) + \Pr(B) - 2\Pr(F \cap B)$

$= 0.65 + 0.39 - 2(0.25)$

$= 0.54$

c) $\Pr(F) = \Pr(F \cap A) + \Pr(F \cap B) + \Pr(F \cap C)$

$0.65 = 0.25 + 0.25 + \Pr(F \cap C)$

$\Pr(F \cap C) = 0.65 - 0.25 - 0.25$

$= 0.15$

Example 2.

Number of outcomes= 2^4=16

$3\ flips = \frac{3}{8}$ $\Pr(1\ head\ in\ 4\ flips) = \frac{4}{16} = \frac{1}{4}$

$THTT, HTTT, TTHT, or\ TTTH$

Example 3.

$A = \{(5,1)(4,2)(3,3)(2,4)(1,5)\}$

$\Pr(sum\ 6) = \frac{5}{36}$ or 0.139 or 13.9%

Example 4.

$B = \{(1,3)(1,4)(2,3)(2,4)(3,3)(3,4)(4,3)(4,4)(5,3)(5,4)(6,3)(6,4)\}$

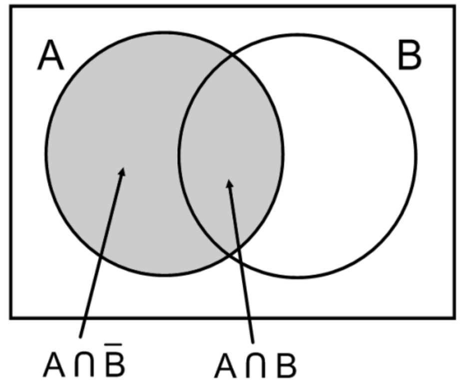
$\Pr(A) = \frac{5}{36}$ $\Pr(B) = \frac{12}{36}$

$A \cap B' = sum\ 6\ and\ not\ 3\ or\ 4\ on\ second\ dice$

$\Pr(A \cup \overline{B}) = \Pr(A) + \Pr(B') - \Pr(A \cap B')$

$= \frac{5}{36} + \left(1 - \frac{12}{36}\right) - \frac{3}{36} = \frac{5}{36} + \frac{24}{36} - \frac{3}{36} = \frac{26}{36} = \frac{13}{18}$=0.72 or 72%

Example 5.



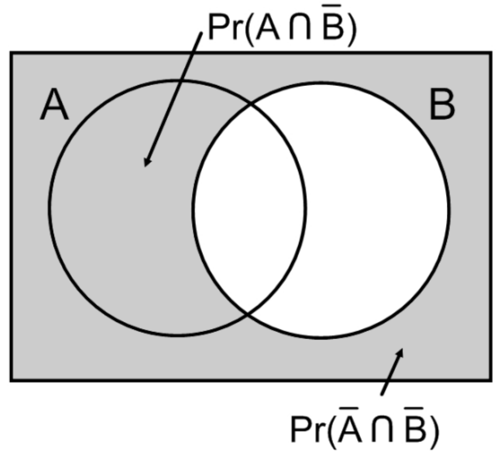
a) $\frac{\binom{13}{2}}{\binom{52}{2}} = 0.0588$

b) $\frac{\binom{13}{1}\binom{4}{2}}{\binom{52}{2}} = 0.0588$

Example 6.

I is false, it should be equal to Pr(A).
II is false, it should be equal to Pr(A).
The answer is D).

Example 7.



All shaded except for B $\therefore 1 - \Pr(B)$
The answer is C.

Example 8.

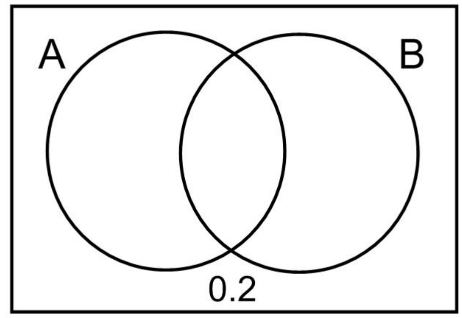
$\Pr(A \cup B) = 1 - \Pr(A' \cap B') = 1 - 0.2 = 0.8$
$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$0.8 = 0.2 + 0.6 - \Pr(A \cap B)$
$\therefore \Pr(A \cap B) = 0$

Since $\Pr(A \cap B) = 0$, A and B are mutually exclusive. (disjoint)

Example 9.

Event	Probability
a	$3x$
b	x
c	
d	0.5 (total of c and d)
e	$2(3x) = 6x$
Total	1

$$3x + x + 0.5 + 6x = 1$$
$$10x = 0.5$$
$$x = 0.05$$
$$\therefore \Pr(a) = 3(0.05) = 0.15$$



Example 10.

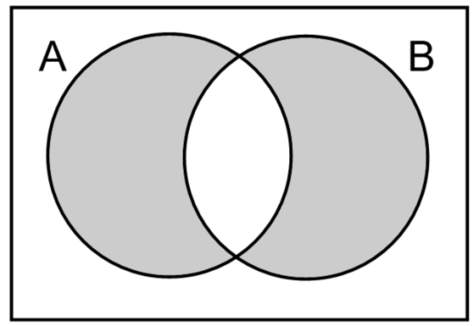
$$\Pr(A \cap B) = \Pr(A) \times \Pr(B) \quad since\ independent$$
$$= 0.32 \times (1 - 0.25)$$
$$= 0.24 \quad or \quad 24\%$$

Example 11.

$A = airplane \quad B = Bus$
$\Pr(A) = 0.50 \quad \Pr(B) = 0.35$
$\Pr(A\ or\ B) = \Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$= \Pr(A) + \Pr(B) - \Pr(A) \times \Pr(B)$
$= 0.50 + 0.35 - 0.50 \times 0.35 = 0.675$

Example 12.

$\Pr(A) + \Pr(B) - 2\Pr(A \cap B)$
$= 0.50 + 0.35 - 2(0.175)$
$= 0.50$



Example 13.
$\Pr(A \cap B) = 0$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - 0$
$0.85 = 0.4 + \Pr(B)$
$\therefore \Pr(B) = 0.45$

$\Pr(B \cap C) = \Pr(B) \times \Pr(C)$
$\Pr(B \cup C) = 0.75$
$\Pr(B \cup C) = \Pr(B) + \Pr(C) - \Pr(B \cap C)$
$0.75 = 0.45 + \Pr(C) - 0.45 \times \Pr(C)$
$0.30 = 1\Pr(C) - 0.45\Pr(C)$
$0.30 = 0.55\Pr(C)$
$\therefore \Pr(C) = 0.545$

Example 14.

a) $\Pr[(A \cup B) \cap C] = \Pr(A \cup B) \times \Pr(C)$
$= [\Pr(A) + \Pr(B) - \Pr(A) \times \Pr(B)] \times \Pr(C)$ since they are independent
$= [0.4 + 0.3 - (0.4)(0.3)] \times 0.7 = 0.406$
b) $\Pr[(A \cap B) \cup C] = \Pr(A \cap B) + \Pr(C) - \Pr[(A \cap B) \cap C]$
$= \Pr(A \cap B) + \Pr(C) - \Pr(A \cap B) \times \Pr(C)$
$= 0.4(0.3) + 0.7 - 0.4(0.3)(0.7) = 0.736$

Example 15.
Pr(at least one club)= 1- Pr(no clubs)
$=1 - \left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = 1 - \frac{9}{16} = \frac{7}{16}$

H1. $\Pr(A) = 1 - \Pr(O) - \Pr(B) - \Pr(AB) = 1 - 0.50 - 0.20 - 0.05 = 0.25$. The answer is (c).

H2. Pr(both aces)=$\frac{4}{52} \times \frac{3}{51}$= or do choose ie. $\frac{\binom{4}{2}}{\binom{52}{2}}$ =0.004525

H3. $\Pr(A \cap B) = \Pr(A) \times \Pr(B) = 0.2(0.3) = 0.06 \neq 0$, so a) is true.
Since A, B are independent
Pr(A or B)=Pr(A) + Pr(B) - 0.06
=0.2 + 0.3 - 0.06
=0.44

So, a) is true since they are independent...b) is true

To check c)...$\Pr(A' \cap B') = 1 - \Pr(A \cup B) = 1 - 0.44 = 0.56$

The answer is d). only a) and b) are true.

H4.Let E denote the event that at least one of the four mosquitoes was a carrier of the virus.
Then E' denotes the event that none of the four mosquitoes was a carrier of the virus.
Since each mosquito has a 90% of not being a carrier of the virus,

$$Pr(\ E') = (0.90)^4 = 0.6561.$$

Therefore $Pr(E) = 1 - Pr(E') = 1 - (0.90)^4 = 0.3439 = 34.39\%.$

H5. The probabilities of drawing 1 red ball, 1 green ball, or 1 yellow ball are

$$\Pr(R) = \frac{5}{10}, \qquad \Pr(G) = \frac{3}{10}, \qquad \Pr(Y) = \frac{2}{10},$$

respectively.
The probabilities of drawing 2 red balls, 2 green balls, or 2 yellow balls are

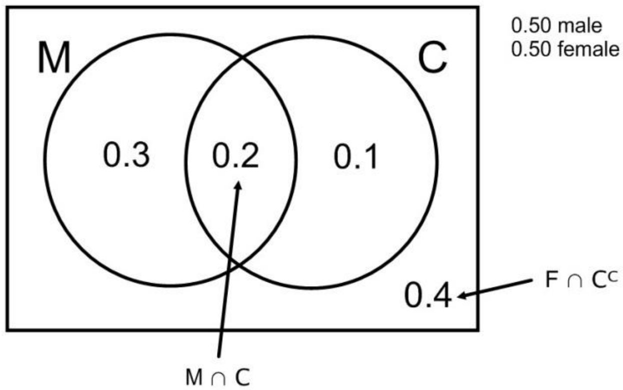
$$\Pr(RR) = \left(\frac{5}{10}\right)^2, \qquad \Pr(GG) = \left(\frac{3}{10}\right)^2, \qquad \Pr(YY) = \left(\frac{2}{10}\right)^2,$$

respectively.
The probability of drawing 2 balls of the same colour is therefore

$$\Pr(RRorGGorYY) = \left(\frac{5}{10}\right)^2 + \left(\frac{3}{10}\right)^2 + \left(\frac{2}{10}\right)^2 = 0.25 + 0.09 + 0.04 = 0.38.$$

H6. If 30% have a college degree and 20% of men have a college degree, then 10% of the women have a college degree

Pr(female and college degree)= 0.10....female without college would be 0.4, if they asked!



H7. The probability of *not* catching a fish each time you cast your line is $1-\frac{1}{4}=\frac{3}{4}$.

The probability of *not* catching a fish on the first two attempts is $(\frac{3}{4})^2=\frac{9}{16}$.

The probability of catching at least one fish within the first two attempts is thus $1-\frac{9}{16}=\frac{7}{16}$.

The answer is (b).

H8. Pr(F)=0.40 and Pr(N)=0.30, Pr(F and N)=0.20

Pr(F or N)= Pr(F) + P(N) – Pr(F and N)=0.40+0.30-0.20=0.50

H9.

a) Pr(40-49)=(10+15+50+70)/400
=145/400=0.3625

b) 50/400

c)145+55/400= 200/400=0.5

d) 15+10/400= 25/400=0.0625

e) 60+30/400=90/400=0.225

H10.
BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG are all the possibilities.

Pr(exactly 2 girls)=3/8=0.375

H11.
Pr(sum greater than 10)=Pr(sum 11 or 12)=Pr{(5,6)(6,5)(6,6)}=3/36=1/12

H12.
a) Yes, they are disjoint as you can't be underweight and obese at the same time...you can only belong to one category

b) Pr(D)= 1 - 0.02 - 0.39 - 0.35=0.24

H13. Pr(red)=26/52
Pr(face card)=12/52

H14. Since A and B are independent...
Pr(A and B)=Pr(A)Pr(B)= 0.2(0.5)=0.10

H15. The following table shows the distribution of blood types in 100 people.

	O	A	B	AB	Total
Rh Positive	39	35	8	4	86
Rh Negative	6	5	2	1	14
Total	45	40	10	5	100

a) If one person is randomly selected, find the probability they have AB blood type

5/100=0.05

b) If one person is randomly selected, find the probability they are O blood type or Rh negative.
Pr(O or Rh^-)= Pr(O) + Pr(Rh^-) - Pr(O and Rh^-)
=45/100 + 14/100 - 6/100
=53/100
=0.53

c) If one person is randomly selected, find the probability they are A blood type and Rh positive.

35/100=0.35

H16.

$\frac{40}{100} \times \frac{39}{99}$=0.158

H17. Suppose events A, B and C are all events in a sample space. You are given that A and B are mutually exclusive and B and C are independent, where Pr(B)=0.1, Pr(A)=0.4 and $\Pr(B \cup C) = 0.6$.

Find each of the following:

a) Pr (C)

$\Pr(B\ or\ C) = \Pr(B) + \Pr(C) - \Pr(B) \times \Pr(C)\ since\ B, C\ are\ independent$
0.6=0.1 + Pr(C) - 0.1Pr(C)
0.5 = 1Pr(C) – 0.1 Pr(C)
Pr(C)=0.5/0.9=5/9

b) Pr $(A \cup B)$=Pr(A) + Pr(B) - 0 since they are mutually exclusive
=0.4+0.1=0.5

H18. The answer is d). If they are mutually exclusive then Pr(A and B)=0...if they were to be independent as well, then Pr(A)xPr(B)=0 and this is impossible since we are told that the probabilities of A and B are non-zero.
Therefore, they would have to be DEPENDENT.

H19. P(A and B)= 1/36 only one outcome since it would be only {(6,1)}
P(AUB)=P(A or B)=P(A)+P(B)- P(A and B)
=6/36 + 6/36 - 1/36
=11/36

H20.
$\frac{2}{10} \times \frac{1}{10} \times \frac{2}{10} = \frac{4}{1000} = 0.004$

H21.
$\Pr(at\ least\ 1\ catch) = 1 - \Pr\ (no\ catch)$
$= 1 - (0.70)(0.60)(0.90)$
$= 0.622\ \ or\ \ 62.2\%$

H22.
$June = 30\ days \qquad July = 31\ days$
$\therefore \frac{30+31}{365} = 0.17\ \ or\ \ 17\%$

H23.

	Snow	No snow	Total
Forecast snow	76	136	212
Forecast no snow	24	264	288
Total	100	400	500

$\therefore correct = \frac{76+264}{500} = \frac{340}{500} = 0.68 \quad or\ 68\%$

H24.

$C = 1 - 0.45 - 0.40 - 0.04$

$\therefore C = 0.11$

$\Pr(same) = \Pr(0,0) + \Pr(A, A) + \Pr(B, B) + \Pr\ (AB, AB)$

$= 0.45^2 + 0.40^2 + 0.11^2 + 0.04^2$

$= 0.3762 \quad or \quad 37.62\%$

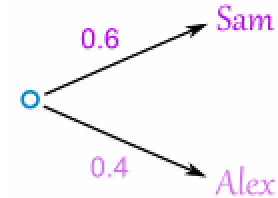
I. Conditional Probability

Example 1.

$$\Pr(F/E) = \frac{\Pr(F \cap E)}{\Pr(E)} = \frac{0.30}{0.40} = \frac{3}{4}$$

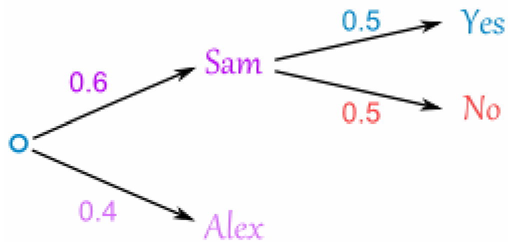
Example 2.

Let's build a tree diagram. First we show the two possible coaches: Sam or Alex:

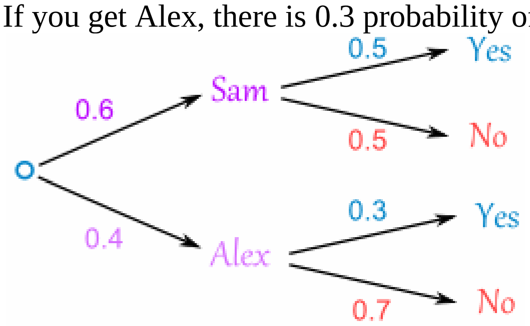


The probability of getting Sam is 0.6, so the probability of Alex must be 0.4 (together the probability is 1)

Now, if you get Sam, there is 0.5 probability of being Goalie (and 0.5 of not being Goalie):



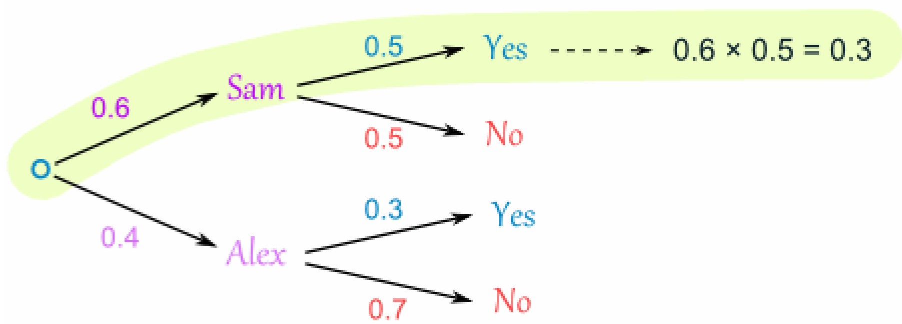
If you get Alex, there is 0.3 probability of being Goalie (and 0.7 not):



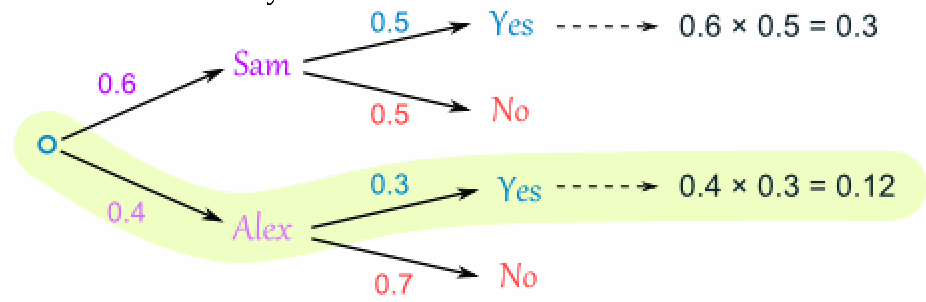
The tree diagram is complete, now let's calculate the overall probabilities. Remember that:

P(A and B) = P(A) x P(B|A)

Here is how to do it for the "Sam, Yes" branch:



(When we take the 0.6 chance of Sam being coach and include the 0.5 chance that Sam will let you be Goalkeeper we end up with an 0.3 chance.)
But we are not done yet! We haven't included Alex as Coach:



A 0.4 chance of Alex as Coach, followed by a 0.3 chance gives 0.12
And the two "Yes" branches of the tree together make:
0.3 + 0.12 = **0.42 probability** of being a Goalkeeper today

$$\Pr(G) = 0.6(0.5) + 0.4(0.3) = 0.42$$

Probability Tree Diagrams. (n.d.). Retrieved May 28, 2022, from https://www.mathsisfun.com/data/probability-tree-diagrams.html

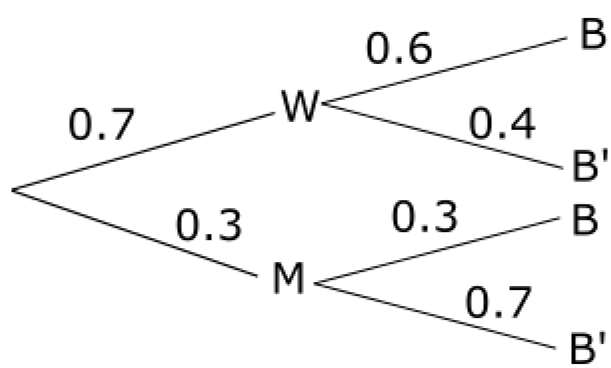
Example 3.

$$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\frac{2}{5} = \frac{\Pr(A \cap B)}{\frac{1}{2}}$$

$$\therefore \Pr(A \cap B) = \frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$$

$$\Pr(B/A) = \frac{\Pr(A \cap B)}{\Pr(A)} = \frac{\frac{1}{5}}{\frac{1}{3}} = \frac{1}{5} \times \frac{3}{1} = \frac{3}{5}$$

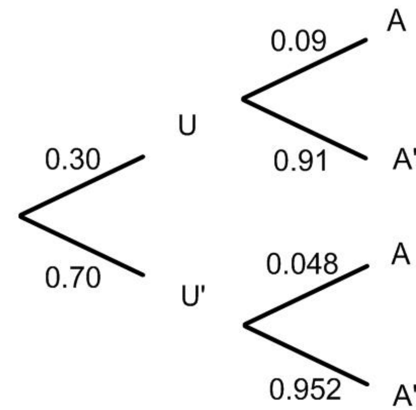


Example 4.

a) $\Pr(B') = 0.70(0.40) + 0.30(0.70)$
$= 0.28 + 0.21$
$= 0.49$

b) $\Pr(W'/B) = \frac{\Pr(W' \cap B)}{\Pr(B)} = \frac{0.30(0.30)}{1-0.49} = 0.176$

Example 5.

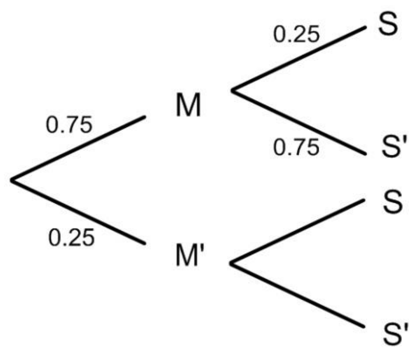


$u = under\ 25$
$P(U) = 0.30\ \ under\ 25$
$\Pr(U/A) = \frac{\Pr\ (U \cap A)}{\Pr\ (A)}$

$= \frac{0.3 \times 0.09}{0.3 \times 0.09 + 0.7 \times 0.048}$

$= 0.446$

Example 6.



a) $0.75 \times 0.25 = 0.1875$

b) $\Pr(M/S') = \frac{\Pr\ (M \cap S')}{\Pr\ (S')}$
$= \frac{0.75(0.75)}{0.80}$
$= 0.70$

Example 7.

$\Pr(x > 3/x \leq 8) = \frac{\Pr(x>3\ and\ x \leq 8\ )}{\Pr(x \leq 8)} = \frac{0.15+0.2+0.1+0.05+0.05}{0.9}$
$= \frac{0.55}{0.90}$ =0.61

Example 8.

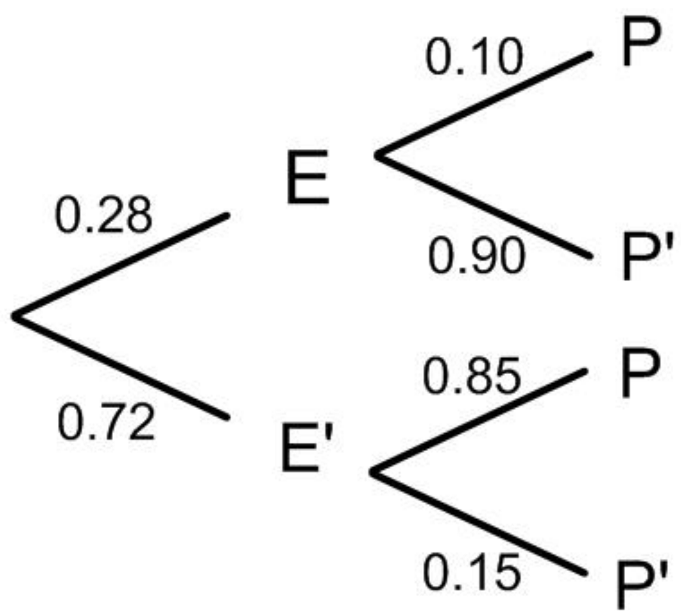
I) True
II) $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)\quad \therefore false$
III) $\Pr(A/B') = \Pr(A)\quad \therefore false$
The answer is C.

Example 9.

$\Pr(A \cap B) = \Pr\ (A) \times \Pr\ (B)$ so A. is true
$\Pr(A/B) = \frac{\Pr\ (A \cap B)}{\Pr\ (B)}\quad cross\ multiply$
$\therefore \Pr(A \cap B) = \Pr\ (B) \times \Pr\ (A/B)$ so C. is true.
$\therefore All\ are\ true\ \ so\ the\ answer\ is\ D.$

Example 10. Let E= emits excessive pollutants and let P= passes the test for emissions

$\text{Pr}\ (E/P') = \frac{\text{Pr}(E \cap P')}{\text{Pr}\ (P')} = \frac{0.28(0.90)}{0.28(0.90)+0.72(0.15)} = 0.70$



Example 11.

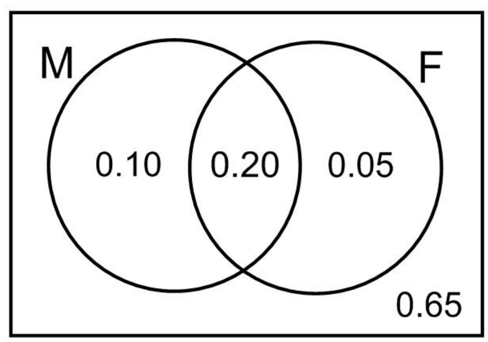
Let M= got an A on the midterm and let F= got an A on the final exam

a) Pr(M)=0.30

Pr(F)=0.25

$\text{Pr}(M \cap F) = 0.20$

We want to find $\text{Pr}(F'/M) = \text{Pr}\ (F' \cap M)/\ \text{Pr}\ (M)$



We aren't given any conditional probabilities, so just draw a Venn diagram to solve this

$\text{Pr}(F'/M) = 0.10/0.30 = 1/3$

b)$\text{Pr}(M' \cap F') = 1 - \text{Pr}(M \cup F) = 1 - 0.35 = 0.65$ using the Venn diagram above

Example 12.

a) $\Pr\left({}^{B3}/_{A2}\right) = \frac{\Pr\,(B3\cap A2)}{\Pr\,(A2)} = \frac{0.06}{0.25} = 0.24$

b) $\Pr(B1) = \Pr(A1 \cap B1) + \Pr(A2 \cap B1) + \Pr\,(A3 \cap B1)$
$= 0.05 + 0.09 + 0.13 = 0.27$

c) $\Pr(A2 \cup B1) = \Pr(A2) + \Pr(B1) - \Pr\,(A2 \cap B1)$
$= 0.25 + 0.27 - 0.09 = 0.43$

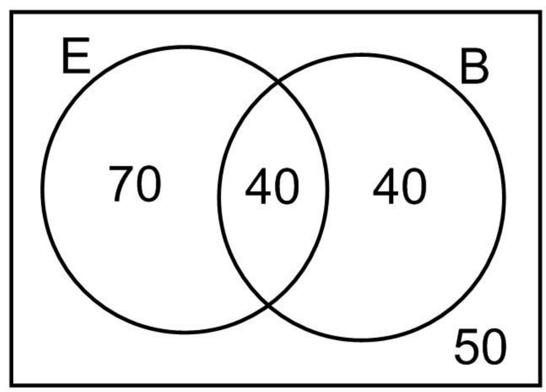
d) Pr(A2∩ $B1'$)
$= \Pr(A2) - \Pr\,(A2 \cap B1)$
$= 0.25 - 0.09$
$= 0.16$

I1.

$\text{n}(E \cup B) = 200 - 50 = 150$

Pr(E)=110/200=11/20
Pr(B)=80/200=2/5

$\text{n}(E \cup B) = \text{n}(E) + \text{n}(B) - \text{n}(E \cap B)$
150=110 +80 – $n(E \cap B)$
$n(E \cap B) = 40$



$\Pr\big((E' \cap B')\big) = \frac{50}{200} = 1/4$

a) Pr(E/B)=$\frac{\Pr\,(E\ and\ B)}{\Pr\,(B)} = \frac{40/200}{80/200} = 1/2$

b) Pr(E or B but not both)= 70+40/200=110/200=11/20
NOTE: you don't include the middle of the Venn

I2.

$\Pr(\text{A/B}) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{0.2}{0.6} = 1/3$

I3.

$\Pr\,(E/F) = \frac{\Pr\,(E\cap F)}{\Pr\,(F)}$

$2/3 = \frac{Pr\,(E\cap F)}{1/3}$

$\Pr(E \cap F) = 2/9$

The answer is a).

I4.

Pr (E/F)=$\frac{\Pr(E \cap F)}{\Pr(F)}$

$$0.40 = \frac{0.2}{\Pr(F)}$$

Pr(F)=1/2

I5.
If A and B are independent, $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$=0.30x0.20=0.06
Pr(AUB)=0.3 + 0.2 - 0.06 = 0.44

C20. They are mutually exclusive, so there is no overlap of circles
Pr(B) = 1 - 0.3 - 0.25 = 0.45

The answer is b).

I6. They are mutually exclusive, so there is no overlap of circles
Pr(B) = 1 - 0.3 - 0.25 = 0.45

The answer is b).

I7.

		Smoking Status		
		Nonsmoker	*Moderate Smoker*	*Heavy Smoker*
Hypertension Status	*Hypertension*	21	36	30
	No Hypertension	48	26	19

(a) What is the probability that a randomly selected individual is experiencing hypertension?

$$\Pr(\text{hypertension}) = \frac{\#\text{ with hypertension}}{\text{total }\#} = \frac{21+36+30}{180} = \frac{87}{180} \approx 0.48$$

(b) Given that a heavy smoker is selected at random from this group, what is the probability that the person is experiencing hypertension?

$$\Pr(\text{hypertension} \mid \text{heavy smoker}) = \frac{\Pr(\text{hypertension} \cap \text{heavy smoker})}{\Pr(\text{heavy smoker})} = \frac{30}{30+19} \approx 0.61$$

(c) Are the events "hypertension" and "heavy smoker" independent? Give supporting calculations.

Since $\Pr(\text{hypertension} \mid \text{heavy smoker}) = \frac{30}{49} \neq \frac{87}{180} = \Pr(\text{hypertension})$, the two events are *not* independent.

I8. **(a)** Are the events *A* and *B* disjoint? Explain.
Yes. They are disjoint because an adult cannot have a college level education and have his highest level of education be secondary.

(b) Are the events *A* and *C* disjoint? Explain.
No. They are not disjoint since females can have a college level education.

(c) What is the probability that an adult selected at random either has a college level education or is female?
Pr(college or female) = Pr(college) + Pr(female) - Pr(college and female)

$$= \frac{22+17}{200} + \frac{45+50+17}{200} - \frac{17}{200} = \frac{39}{200} + \frac{112}{200} - \frac{17}{200} = \frac{134}{200} = 0.67.$$

(d) What is the probability that an adult selected at random has a college level education given that the adult is a female?

$$\Pr(\text{college} \mid \text{female}) = \frac{\Pr(\text{college and female})}{\Pr(\text{female})} = \frac{\frac{17}{200}}{\frac{112}{200}} = \frac{17}{112} \approx 0.15$$

(e) Are the events *A* and *C* independent?

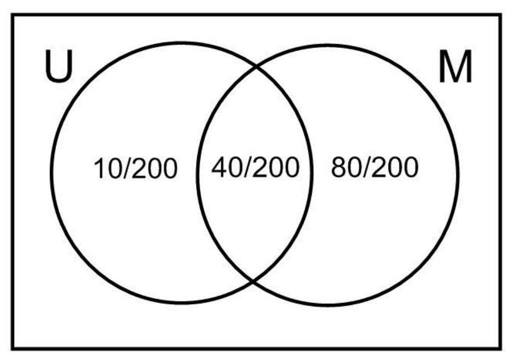
$$\Pr(A) = \frac{22+17}{200} = \frac{39}{200}; \qquad \Pr(C) = \frac{45+50+17}{200} = \frac{112}{200} = \frac{14}{25};$$

$$\Pr(A \cap C) = \frac{17}{200} = 0.085; \qquad \Pr(A)\Pr(C) = \frac{39}{200} \cdot \frac{14}{25} = \frac{273}{2500} = 0.1092;$$

Since $\Pr(A \cap C) \neq \Pr(A)\Pr(C)$, the events *A* and *C* are not independent.

I9.
Pr(Female and lower division)=1 - 130/200 = 70/200

Pr(lower division /female)=$\frac{\Pr(both)}{\Pr(female)} = \frac{70/200}{80/200} = \frac{70}{80} = 0.875$



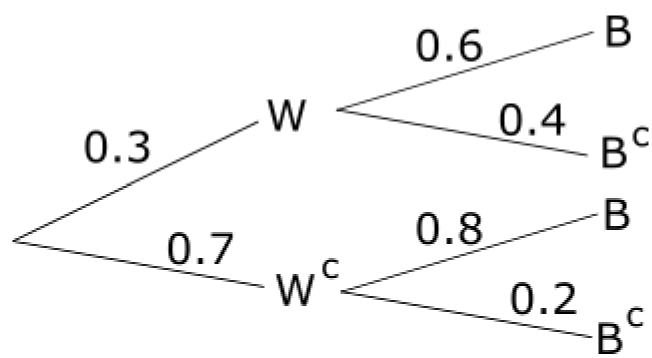
I10.
Pr(fail stop/not signal)=$\frac{\Pr(fail\ stop\ and\ not\ signal)}{\Pr(not\ signal)} = \frac{0.10}{0.15} = \frac{10}{15} = \frac{2}{3}$=0.67

I11.

Pr(B)= 0.3(0.6) + (0.70)(0.80)= 0.18 + 0.56 = 0.74

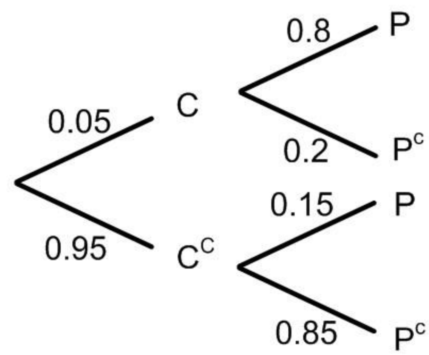
I12. Use Pr(B)=0.74 from I11.

Pr(W^C/B)=$\frac{\Pr(W^c \cap B)}{\Pr(B)} = \frac{0.7(0.8)}{0.74} = \frac{56}{74}$



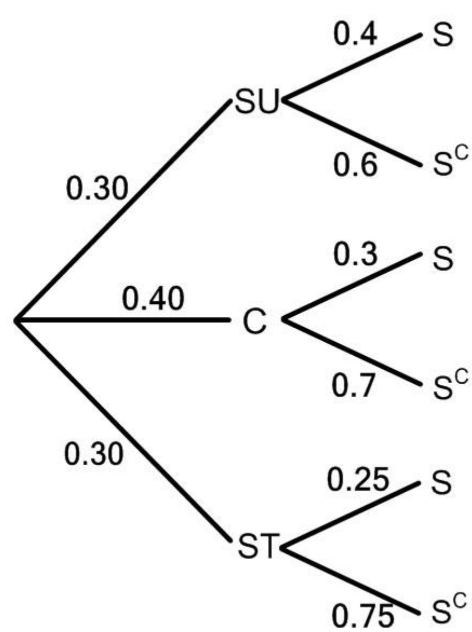
I13. **NOTE that** $C^C = C' = free\ of\ cancer$

Draw a Tree diagram



Pr(C^C/P)=$\frac{\Pr(C^C \cap P)}{\Pr(P)} = \frac{0.95(0.15)}{0.95(0.15)+0.05(0.80)}$=0.781

I14. Pr(ST/S)=$\frac{\Pr(ST \cap S)}{\Pr(S)} = \frac{0.30(0.25)}{0.3(0.25)+0.4(0.3)+0.3(0.4)}$=0.238



I15. The following two-way table shows the age and sex of all undergraduate university students at a particular university.

Age Group	Female	Male	Total
15-17 years	200	250	450
18-20	3000	3500	6500
21-26	2000	2500	4500
27-34	800	900	1700
35+	500	300	800
Total	6500	7450	13950

Let A= student chosen at random is female
B= student chosen at random is over 26 years old

Find each of the following:

a) Pr(A and B)
=1300/13950=0.093 or 9.3%

b) Pr(A/B) =Pr(female/over 26)= only look at those over 26 and circle number of females
$=\frac{800+500}{1700+800}=\frac{1300}{2500}=0.52\ or\ 52\%$

c) Pr(B/A)= Pr(over 26/female)= only look at females and circle those who are over 26
$=\frac{800+500}{6500}=\frac{1300}{6500}=0.2\ or\ 20\%$

d) Pr (A'/B)= Pr(not female/over 26 years old)= only look at people over 26 years old and circle the men
$=\frac{900+300}{1700+800}=\frac{1200}{2500}=0.48\ or\ 48\%$

I16. If $\Pr(A \cup B)=0.7\ and\ \Pr(A)=0.5\ and\ \Pr(B^C)=0.6,\ find\ \Pr(A \cap B)$.
Pr(B)=1 - 0.60= 0.40

$PrA \cup B)=\Pr(A)+\Pr(B)-\Pr\ (A \cap B)$

0.70 = 0.50 + 0.40 - Pr$(A \cap B)$

Pr(A$\cap B)=0.20$

I17.

Pr(A U B)=Pr(A) + Pr(B) - Pr(A and B)
0.70=0.4 + 0.5 - Pr(A and B)
Pr(A and B)=0.2

Pr(B/A)=$\frac{\Pr\ (A\ and\ B)}{\Pr\ (A)} = \frac{0.2}{0.4} = 0.5$

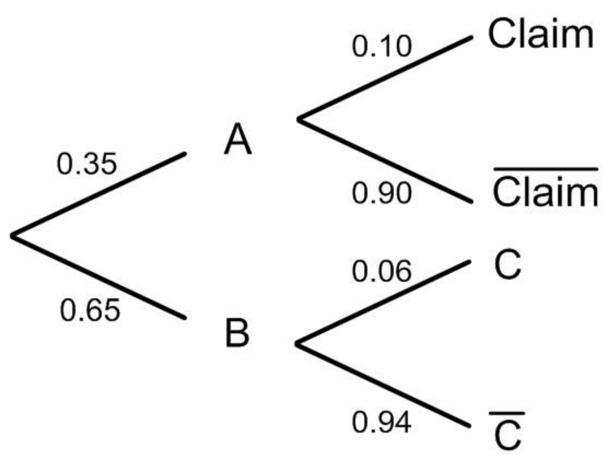
I18.

$\Pr\left(\frac{x<7}{x>3}\right) = \frac{\Pr\ (x<7\ \ and\ x>3)}{\Pr\ (x>3)}$
$= \frac{0.15+0.05+0.15}{1-0.05-0.10-0.20}$
$= \frac{0.35}{0.65}$

$= 0.538\ \ or\ \ 53.8\%$

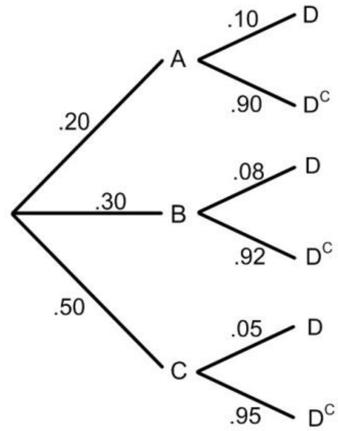
I19.

$\Pr(A/C) = \frac{\Pr\ (A\cap C)}{\Pr\ (C)} = \frac{0.35(0.10)}{0.35(0.10)+0.65(0.06)} =$
$0.473\ \ or\ \ 47.3\%$



J. Bayes' Theorem

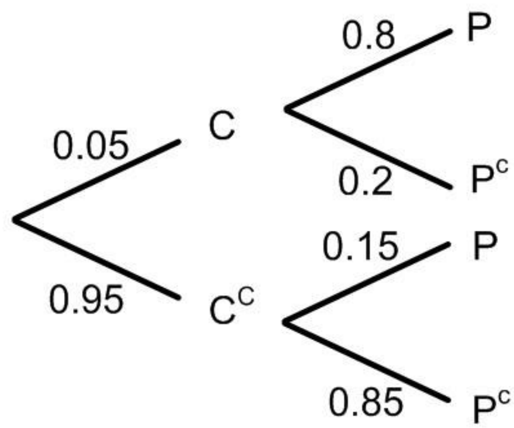
Example 1.



Pr(A/D)=$\frac{\Pr(D/A)\Pr(A)}{\Pr(D)} = \frac{0.10(0.20)}{0.20(0.10)+0.30(0.08)+0.50(0.05)} = 0.2899$

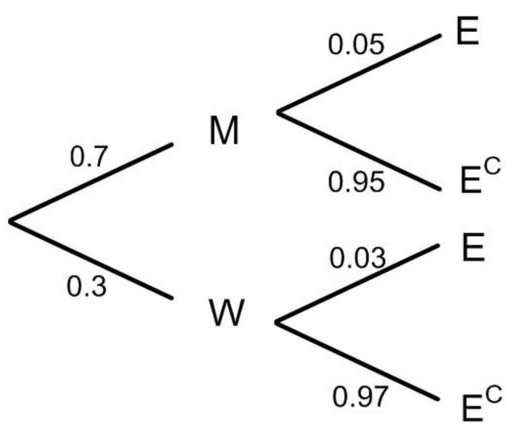
J1.
Draw a Tree diagram

Pr(C'/P)=$\frac{\Pr(C' \text{ and } P)}{\Pr(P)} = \frac{(0.95)(0.15)}{0.95(0.15)+0.05(0.80)} = 0.7808$



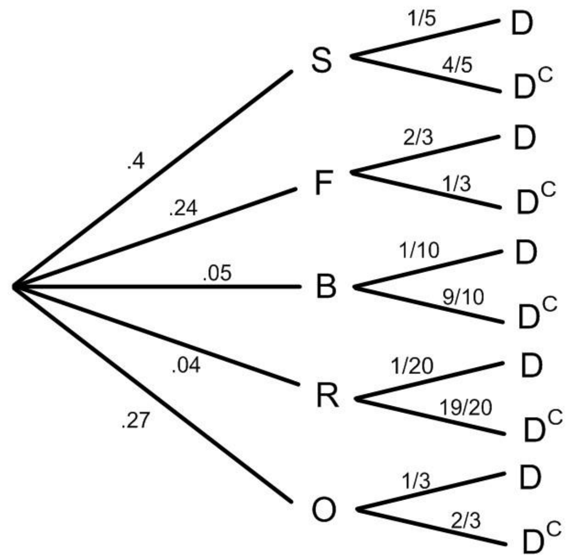
J2.
Let E=earn more than $40000

Pr(W/E)=$\frac{\Pr(W \text{ and } E)}{\Pr(E)} = \frac{0.30(0.03)}{0.3(0.03)+0.70(0.05)}$=0.205



J3.

a) Had a fall



$$\Pr(F/D)=\frac{\Pr(F \text{ and } D)}{\Pr(D)}=\frac{0.24(\frac{2}{3})}{0.4\left(\frac{1}{5}\right)+0.24\left(\frac{2}{3}\right)+0.05\left(\frac{1}{10}\right)+0.04\left(\frac{1}{20}\right)+0.27(\frac{1}{3})}=0.475$$

b) Stung by a bee.

$$\Pr(B/D)=\frac{\Pr(B\cap D)}{\Pr(D)}=\frac{0.05(\frac{1}{10})}{same\ as\ a)}=0.0148$$

J4.

Let M be the event that a marking mistake was made.

$$\begin{aligned}\Pr(M) &= \Pr(A\cap M)+\Pr(B\cap M)+\Pr(C\cap M)\\ &= \Pr(M\mid A)\Pr(A)+\Pr(M\mid B)\Pr(B)+\Pr(M\mid C)\Pr(C)\\ &= (0.1)\cdot(0.50)+(0.2)\cdot(0.30)+(0.3)\cdot(0.20) = 0.17.\end{aligned}$$

Therefore, $\Pr(A\mid M) = \dfrac{\Pr(A\cap M)}{\Pr(M)} = \dfrac{\Pr(M\mid A)\Pr(A)}{\Pr(M)} = \dfrac{(0.1)\cdot(0.5)}{0.17} = \dfrac{0.05}{0.17} = 0.29$.

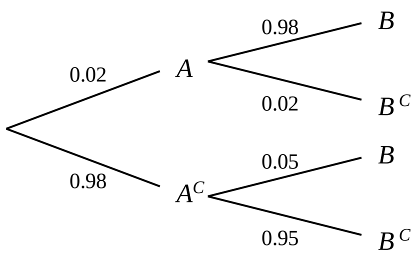
J5.

(a) Since 95% of non-infected dogs indicate no infection, therefore 5% of the non-infected dogs will indicate an infection.

(b)

Let $A =$ "dog is infected",
and $B =$ "test indicates infection".

$$\Pr(A \mid B) = \frac{\Pr(A and B)}{\Pr(B)}$$



$$= \frac{\Pr(B \mid A)\Pr(A)}{\Pr(B \mid A)\Pr(A) + \Pr(B \mid \overline{A})\Pr(\overline{A})} = \frac{(0.98)\cdot(0.02)}{(0.98)\cdot(0.02) + (0.05)\cdot(0.98)}$$

$$= \frac{0.0196}{0.0196 + 0.0490} = \frac{0.0196}{0.0686} = \frac{2}{7} \approx 0.2857 .$$

(c)

$$\Pr(\overline{A} \mid B) = 1 - \Pr(A \mid B) = 1 - \frac{2}{7} = \frac{5}{7} \approx 0.7143$$

K. Permutations and Combinations

Example 1. a) 6! = 720 b) $\frac{11!}{2!2!2!}$ since there are 2 repeating T's and 2 repeating M's and two repeating A's.

Example 2. $C_{10}^{3} \times 3! = P_{10}^{3} = \binom{10}{3} \times 3!$
$= \frac{10!}{3!7!} \times 3! = \frac{10!}{7!} = 720$

Example 3.
a) put all the dog photos together in one spot...there are 4 units to arrange and then multiply by another 4! for the dog photos to be arranged in their own group

The answer is 4!4!=576

b) put the dog photos together and the cat photos together...then, multiply be 3! to arrange the cat photos in their own group and 4! to arrange the dog photos in their own group

The answer is 2!3!4! or 288

Example 4. $P_{10}^{6} = \frac{10!}{(10-6)!} = \frac{10!}{4!} = 151{,}200$

Example 5.$\binom{12}{3\ \ 3\ \ 3\ \ 3} = \frac{12!}{3!3!3!3!} = 369600$

Example 6. a) put the M first...8 other letters, so the answer is 8!

b) put the vowels together in one spot...7 units to arrange and then multiply by 3! for the A, I and U to arrange in their own group of 3
answer is 7!3!

c) total ways - # ways M , A are together
=9! -8!2!

Example 7.
You can either have an appetizer or a soup, but not both, so do two cases 1. Appetizer and no soup and 2. Soup but no appetizer
$6 \times 8 \times 5 \times 7 + 3 \times 8 \times 5 \times 7 = 2520$

Example 8.
There are 12 choices for each prize, since you are putting the names back in each time, so $12^3 = 1728$

b) If you don't put the names back in, you have $12 \times 11 \times 10 = 1320$ combinations

Example 9. Place A and M together to start the circle, there are 4 people left, so 4! To arrange them and then A and M can switch places, so 4!2!

There are 4!2! ways for Alis and Meghan to be beside one another in a circle.

b) For them apart, we do Total ways – number of ways they are together
=5! -4!2!

K1. $8! = 40320$

K2. $\binom{25}{3} = C_3^{25} = \frac{25!}{3!22!} = 2300$

K3. $P_3^5 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{120}{2} = 60$

K4. Arrange =permutations
$P_4^{10} = \frac{10!}{(10-4)!} = \frac{10!}{6!} = 5040$

K5. 5R, 3M, 2H

a) Pr(1R) = $\frac{\binom{5}{1}\times\binom{5}{2}}{\binom{10}{3}} = 0.417$

b) Pr(at least 1R) = 1-Pr(no R)

$$= 1 - \frac{\binom{5}{0}\times\binom{5}{3}}{\binom{10}{3}} = 1 - 0.08\overline{3} = 0.92$$

K6. Pr(1 of each) = $\frac{\binom{5}{1}\times\binom{3}{1}\times\binom{2}{1}}{\binom{10}{3}} = 0.25$

K7. 3P, 5B, 3Bl $\rightarrow$ 3

Pr(1 of each) = $\frac{\binom{3}{1}\times\binom{5}{1}\times\binom{3}{1}}{\binom{11}{3}} = 0.273$

K8. 8R, 4B, 3P, 5O → 5

a) Pr(at least 2P) = 2P or 3P since there is no replacement and there are only 3 purple or 1-Pr(0 purple) – Pr(1 purple)

$= 1-\frac{\binom{3}{0}\times\binom{17}{5}}{\binom{20}{5}} - \frac{\binom{3}{1}\times\binom{17}{4}}{\binom{20}{5}} = 1 - 0.399 - 0.464 = 0.14$

b) Pr(2 red, 1 blue, 1 purple, 1 orange)$= \frac{\binom{8}{2}\times\binom{4}{1}\times\binom{3}{1}\times\binom{5}{1}}{\binom{20}{5}}$

$= \frac{1680}{15504} = 0.108$

K9. 5M, 6W → 4

a) = 3 women or 4 women

$= \binom{6}{3}\binom{5}{1} + \binom{6}{4} = 100 + 15 = 115$

b) At least 1 man= Total – no men

$= \binom{11}{4} - \binom{5}{0}\times\binom{6}{4} = 330 - 15 = 315$

K10. a) $\binom{52}{5} = 2598960$

b)$\binom{13}{5} = 1287$

c)$\binom{4}{1}\times\binom{13}{5} = 5148$

$\uparrow$ *choose* 1 *of* 4 *suits*

K11. 8M, 7W

\# ways at least 1 women = Total ways - # ways no women

$=\binom{15}{5} - \binom{8}{0}\times\binom{7}{5}$

= 3003 – 21

= 2982

K12. 4M, 5H

$\binom{9}{3} = 84$

K13. Pr (all H) = $\frac{\binom{5}{3}}{\binom{9}{3}} = \frac{10}{84} = 0.119$

K14. - permutation since you want to assign duties

$\binom{15}{3}\times 3! \quad or \quad {}^{15}_{3}P = 2730 \quad type\ 15\ nPr\ 3$

K15. Total – together = 6! – 5!2!

SA __ __ __ __

K16. 7 girls, 6 boys

a) To alternate, you must start with girls since it you start with boys, you would have 2 girls left at the end and it wouldn't be alternating.
GBGBGBGBGBGBG = 7!6!

b) 6 girls, 6 boys
GBGBGBGBGBGB OR BGBGBGBGBGBG
= 6!6!2! since starting with a boy and starting with a girl
is a different arrangement in a line

K17. $\{1,2,3,4,5\}$ odd $\therefore$ $last\ digit\ has\ 3\ choices$ $\rightarrow 1,3\ or\ 5$
Then, 5 choices for first and second digits since repetition is allowed

$5 \times 5 \times 3$ choices =75 choices

K18. The number of ways D, G part = total ways - # ways D, G together = 10! – 9! 2!

The probability of D,G apart = 1- Pr(D,G together) = $1 - \frac{9!2!}{10!}$

K19. Odd numbers less than 300 can be a 1, 2 or 3 digit number
=_+_ _+_ _ _
2 choices + 4×2 + 2×4×2
The one digit number can be a "1" or a "3" (must be odd)
The two digit number can end in a "1" or a "3" to be odd and since repetition is allowed, there are 4 choices for the first digit.
The three digit number must end in a "1" or "3" to be odd. Then, it must be less than 300
$\therefore$ can only start with a "1" or "2"
$\therefore$ $2 + 8 + 16 = 26\ choices$

K20. If you put Tina and Stephanie together in one spot, you get
TS ___ ___ ___ ___ ___ ___
So, the answer would b 7!2! since 2! Is for TS or ST (they can switch places and still be together)

K21. Place Tina and Steph beside each other to start the circle. There are 6 people left to arrange, so we get a 6!. Then, Tina and Steph can switch spots in 2! ways so the final answer is 6!2!.

L. Random Variables

Example 1.

x	Pr (x)	
1	$\frac{1}{36}$	(1,1)
2	$\frac{3}{36}$	(2,2)(2,1)(1,2)
3	$\frac{5}{36}$	(1,3)(3,1) (2,3)(3,3)(3,2)
4	$\frac{7}{36}$	(1,4)(4,1)(4,3)(3,4)(4,4)(4,2)(2,4)
5	$\frac{9}{36}$	(5,5)(4,5)(3,5)(5,3)(2,5) (5,2)(5,1)(1,5)(5,4)
6	$\frac{11}{36}$	(6,6)(5,6)(6,5)(4,6)(6,4)(6,3)(3,6)(2,6)(6,2)(6,1)(1,6)

Example 2. net winnings, so you take away the $1 you pay from your winnings

X	Pr[X=x]
2-1=1	3/6 (even #)
-2-1 = -3	2/6 (roll 1 or 3)
4-1 = 3	1/6 (roll a 5)

E(X)=1(3/6)+(-3)(2/6)+3(1/6)=$ 0 So, it is a fair game!

Example 3.

x	$P(x)$	x^2
1	0.25	1
3	0.50	9
5	0.25	25
	$\overline{1}$	

$\mu = \sum xP(x) = 1(0.25) + 3(0.5) + 5(0.25) = 3$

$\sigma^2 = \sum x^2P(x) - \mu^2 = 1(0.25) + 9(0.5) + 25(0.25) - 3^2$

$\sigma^2 = 2$

$\sigma = \sqrt{2}$

$= 1.41$

Example 4.

From the previous example x= 1, 3 and 5

X=1 $|1-2|<1<2\ is\ true, so\ we\ add\ 0.25, the\ pr(x=1)$

X=3 $|3-2|<2\ is\ true, so\ we\ add\ 0.50, the\ pr(x=3)$

X=5 $|5-2|<2\ is\ false\ so\ we\ don't\ count\ Pr(x=5)$

So, Pr $|x-2|<2\ is\ 0.25+0.50=0.75$

Example 5.

x	$\Pr(x)$	
-1	2c	$\frac{2}{10} = 0.2$
0	3c	$\frac{3}{10} = 0.3$
1	c	$\frac{1}{10} = 0.1$
2	4c	$\frac{4}{10} = 0.4$
	$\overline{1}$	

2c + 3c + c + 4c = 1
10c = 1
c=1/0 or 0.10

$$\mu = \sum x \Pr(x) = -1(0.2) + 0(0.3) + 1(0.1) + 2(0.4) = 0.7$$
$$\sigma^2 = \sum x^2 \Pr(x) - \mu^2$$
$$= (-1)^2(0.2) + 0^2(0.3) + 1^2(0.1) + 2^2(0.4) - 0.7^2$$
$$= 1.41$$
$$\therefore \sigma = 1.19$$

L1.

X	Pr(X=x)	X^2
-1	3a=3/9	1
0	2a=2/9	0
2	2a=2/9	4
4	2a=2/9	16
TOTAL	1	

3a + 2a + 2a + 2a = 1
a=1/9

a) Pr(X=2)=2/9

b) μ =E(X)=$-1\left(\frac{3}{9}\right) + 0\left(\frac{2}{9}\right) + 2\left(\frac{2}{9}\right) + 4\left(\frac{2}{9}\right) = \frac{9}{9} = 1$

c) $x^2 p(x) =$ E(X^2)=$1\left(\frac{3}{9}\right) + 0\left(\frac{2}{9}\right) + 4\left(\frac{2}{9}\right) + 16\left(\frac{2}{9}\right) = \frac{43}{9}$

$$V(X) = x^2 p(x) - \mu^2 = \frac{43}{9} - (1)^2 = \frac{43}{9} - \frac{9}{9} = \frac{34}{9} = 3.78$$

L2. The value at x=2.2 is 1-0.02-0.05-0.2-0.6-0.1=0.03

$$\mu = \sum xPr(x) = 1.7(0.02) + 1.8(0.05) + 1.9(0.2) + \cdots + 2.2(0.03) = 1.98$$

$\sigma = \sqrt{x^2 \Pr(x) - \mu^2} = \sqrt{1.7^2(0.02) + 1.8^2(0.05) + \cdots + 2.2^2(0.03) - 1.98^2}$
=0.087

L3.

$\mu = 1(0.3) + 2(0.2) + 0(0.5) = 0.3 + 0.4 = 0.7$

L4.

X	Pr(X)	X^2
1	0.3	1
2	0.2	4
0	0.5	0

E(X)= $\mu = 1(0.3) + 2(0.2) + 0(0.5) = 0.3 + 0.4 = 0.7$
$x^2p(x) = E(X^2) = 0.3(1) + 4(0.2) + 0(0.5) = 0.3 + 0.8 = 1.1$

V(X)=E(X^2) - (E(X))^2= $x^2p(x) - \mu^2$
=1.1 – 0.7^2
=1.1-0.49
=0.61

L5.

X	Pr(X)	X^2
-1	$1/2$	1
0	$1/4$	0
1	$1/4$	1

$E(X) = -1 \times \frac{1}{2} + 0 \times \frac{1}{4} + 1 \times \frac{1}{4} = -\frac{1}{2} + \frac{1}{4} = -\frac{1}{4}$
$E(X^2) = x^2p(x) = 1\left(\frac{1}{2}\right) + 0\left(\frac{1}{4}\right) + 1\left(\frac{1}{4}\right) = \frac{3}{4}$

$V(X) = x^2p(x) - \mu^2 = E(X^2) - (E(X))^2 = \frac{3}{4} - \left(\frac{-1}{4}\right)^2 = \frac{12}{16} - \frac{1}{16} = \frac{11}{16}$
$\sigma(X) = \sqrt{\frac{11}{16}}$=0.829

M. Binomial Random Variables

Example 1.

$\binom{50}{20}(0.5)^{20}(0.5)^{30}=\binom{50}{20}(0.5)^{50}=0.0419$...multiplying with same base=add exponents

Example 2.

Pr(at least 1 head)=1-Pr(no heads)= $1-\binom{25}{0}(0.5)^{0}(0.5)^{25}=1-(0.5)^{25}=0.\overline{999}$

Example 3.

$n=10 \quad p=0.25\left(\frac{1}{4}\right) \quad q=0.75 \quad x=8$

$\Pr(x=8)=\binom{10}{8}(0.25)^{8}(0.75)^{2}=0.000386$

$$\Pr(x\geq 8)=\Pr(x=8)+\Pr(x=9)+\Pr(x=10)$$
$$=\binom{10}{8}(0.25)^{8}(0.75)^{2}+\binom{10}{9}(0.25)^{9}(0.75)^{1}+\binom{10}{10}(0.25)^{10}(0.75)^{0}$$

Example 4.

$p=0.60\ (tails) \quad q=0.40(heads) \quad n=10 \quad x=5\ to\ 7\ tails$

$\therefore p\ must\ be\ probability\ of\ tails$

$\Pr(5\leq x\leq 7)=\Pr(x=5)+\Pr(x=6)+\Pr(x=7)$

$=\binom{10}{5}(0.6)^{5}(0.4)^{5}+\binom{10}{6}(0.6)^{6}(0.4)^{4}+\binom{10}{7}(0.6)^{7}(0.4)^{3}$

Example 5.

Pr(at least 1H/at least 2 tails)
=Pr(at least 1H and at least 2 tails/ Pr(at least 2T)

* to have both at least 1H and at least 2 tails...we have all but the cases below
0T,10H
10T, 0H,
1T,9H
On the bottom we have Pr(at least 2 T) =1- Pr(no T) – Pr(1T)

So, if we do 1- these cases, we get the probability we want

$$=\frac{1-\binom{10}{0}(0.5)^{0}(0.5)^{10}-\binom{10}{1}(0.5)^{1}(0.5)^{9}-\binom{10}{10}(0.5)^{10}(0.5)^{0}}{1-\binom{10}{0}(0.5)^{0}(0.5)^{10}-\binom{10}{1}(0.5)^{1}(0.5)^{9}}$$

Example 6.

$n = 30 \quad p = q = 0.50$

$V(x) = npq$

$= 30(0.5)(0.5)$

7.5

Example 7.

$CV = 11\% \qquad \sigma^2 = 26 \qquad binomial\ \ find\ q$

$CV = \frac{\sigma}{\mu} \times 100$

$11 = \frac{\sqrt{26}}{\mu} \times 100$

$11\mu = 509.9$

$\mu = 46.35$

$\therefore \mu = np$

$46.35 = np \quad substitute\ into \quad \sigma^2 = npq$

$26 = 46.35q$

$q = 0.56$ p= 1- 0.56 = 0.44

M1.

n=6
p=1/6 get a 4
q=5/6

$\Pr(X = 2) = \binom{6}{2}(\frac{1}{6})^2(\frac{5}{6})^4 = 0.2009$

M2. n=100
p=0.5 heads
q=0.5 tails

$\Pr(X = 25) = \binom{100}{25}(0.5)^{25}(0.5)^{75}$

M3. p=0.8
q=0.2
n=5

Pr(at most 2)=Pr(X=0) + Pr(X=1) + Pr(X=2)
=$\binom{5}{0}(0.8)^0(0.2)^5 + \binom{5}{1}(0.8)^1(0.2)^4 + \binom{5}{2}(0.8)^2(0.2)^3 = 0.05792$

b) Pr(hit at least 1)= 1 - Pr(X=0)
= 1 - $\binom{5}{0}(0.8)^0(0.2)^5$
=0.99968

M4.
p=0.02 defective
q=0.98 not defective

n=10

$\Pr(X \leq 8) = 0,1,2,3,4,5,6,7 \; or \; 8 = too\ difficult, so\ use\ 1 - \Pr(X=9) - \Pr\,(X=10)$
$= 1 - \binom{10}{9}(0.02)^9(0.98)^1 - \binom{10}{10}(0.02)^{10}(0.98)^0$

$= 1 - \binom{10}{9}(0.02)^9(0.98)^1 - (0.02)^{10}$=1

M5.
a) p=0.8
q=0.2
one graduate = 4 that do graduate
Pr(1 graduate)= $\binom{5}{1}(0.80)^1(0.20)^4$ or $\binom{5}{4}(0.80)^1(0.20)^4$ =0.0064(same answer)

b)Pr(3 not graduate) = Pr(2 do graduate)= $\binom{5}{2}(0.8)^2(0.2)^3 \; or \; \binom{5}{3}(0.8)^2(0.2)^3$

M6. n=10 p=4/5 q=1/5

$\Pr(X \geq 8) = \Pr(X=8) + \Pr(X=9) + \Pr\,(X=10)$
=$\binom{10}{8}\left(\frac{4}{5}\right)^8\left(\frac{1}{5}\right)^2 + \binom{10}{9}\left(\frac{4}{5}\right)^9\left(\frac{1}{5}\right)^1 + \binom{10}{10}\left(\frac{4}{5}\right)^{10}\left(\frac{1}{5}\right)^0$

=0.678

M7. a)

p=1/3
q=2/3
n=6

Pr(no hits)=$\binom{6}{0}\left(\frac{1}{3}\right)^{0}\left(\frac{2}{3}\right)^{6}$=$\left(\frac{2}{3}\right)^{6}$=0.08779

b)

Pr(at least 1 hit)= 1 - Pr(no hits)
=$1-\left(\frac{2}{3}\right)^{6}$=0.9122

c) Pr(X=2)= $\binom{6}{2}\left(\frac{1}{3}\right)^{2}\left(\frac{2}{3}\right)^{4} = 0.329$

Use the following information for #M8-10.

M8.

n=10 Bernoulli
p=ace=4/52 = 1/13
q=12/13

Pr(exactly 1 ace) =$\binom{10}{1}\left(\frac{1}{13}\right)^{1}\left(\frac{12}{13}\right)^{9}$

The answer is b).

M9. Pr(at least 1 heart = 1 - Pr(no hearts)

$= 1-\binom{10}{0}\left(\frac{1}{4}\right)^{0}\left(\frac{3}{4}\right)^{10}$

The answer is d).

M10. Bernoulli with n=10 p=4/52=1/13 four and q=48/52=12/13 (not a four)
Pr(at least one four)= 1 - Pr (no fours)
$1-\binom{10}{0}\left(\frac{1}{13}\right)^{0}\left(\frac{12}{13}\right)^{10}$=1 - $\left(\frac{12}{13}\right)^{10} = 0.55$

M11. n=10 questions p=1/5 correct answer and q=4/5 incorrect answer

Pr(x=5 correct)=$\binom{10}{5}\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^5 = 0.0264$

b) Pr(at least one of five)= 1 - Pr(none of five correct)

=$1-\binom{5}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^5$= 1 - $\left(\frac{4}{5}\right)^5$=0.672

c) $\mu = np = 10(0.2) = 2$ Var=npq=10(0.2)(0.8)=1.6

$sd = \sigma = \sqrt{npq} = \sqrt{10(0.2)(0.8)} = \sqrt{1.6} = 1.265$

M12.

$\mu = 40 = np \quad \sigma = 2 \quad find\ p$

$\sigma^2 = npq \quad substitute$

$2^2 = 40q$

$q = 4/40 = 0.1$

$\therefore p = 1 - 0.1 = 0.9$

M13.

$8, 9\ or\ 10\ (makes)$

$p = 0.70\ (makes) \quad q = 0.30(fails) \quad n = 10$

$\therefore P(8) + P(9) + P(10)$

$= \binom{10}{8}(0.7)^8(0.3)^2 + \binom{10}{9}(0.7)^9(0.3)^1 + \binom{10}{10}(0.7)^{10}(0.3)^0$

N. Poisson Random Variable

Example 1.

$\lambda = \frac{5.2}{hour} \times 1.5 = \frac{7.8}{90\ minutes}$

$x = 5\ \ (in\ 90\ minutes)$

$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$P(5) = \frac{e^{-7.8}7.8^5}{5!} = 0.099$

Example 2.

$\lambda = 4\ customer\ per\ hour \quad \therefore \lambda = 2\ per\ \frac{1}{2} hour$

$P(x \geq 1)\ at\ least\ 1\ in\ \frac{1}{2} hour = 1 - \Pr(x = 0)$

$= 1 - \frac{e^{-2}2^0}{0!}$

$= 0.865$

Example 3.

$\lambda = 1.2\ flaws\ \ (in\ 20m)$

a) $\lambda = 1.2 \times 3 = 3.6\ \ (in\ 60\ m)$

$$\Pr(x = 2) = \frac{e^{-3.6}3.6^2}{2!} = 0.177$$

b) $\lambda = 1.2 \times 2 = 2.4\ \ (in\ 40\ m)$

P(at least 2 flaws)= 1 – Pr(no flaws) – Pr(1 flaw)

$P(x \geq 2) = 1 - \Pr(x = 0) - \Pr(x = 1)$

$= 1 - \frac{e^{-2.4}2.4^0}{0!} - \frac{e^{-2.4}2.4^1}{1!}$

=1-0.0907-0.2177

=0.6916

Example 4.

a) $\lambda = 0.2\ flaws\ per\ square\ foot \times 8 = 1.6 (in\ 8\ square\ feet)$

$P(x \leq 1) = P(0) + P(1) \quad in\ 10\ sq\ ft$

$= \frac{e^{-1.6}1.6^0}{0!} + \frac{e^{-1.6}1.6^1}{1!}$

$= 0.2019 + 0.3230$

$\doteq 0.5249$

b) $n = 15$

$binomial\ and\ poisson$

$x = 5\ with\ no\ flaws$

$\therefore p = poisson = \frac{e^{-1.6}1.6^0}{0!} = 0.2019$(prob. of no flaws)

$$P(x = 5) = \binom{15}{5}(0.2019)^5(0.7981)^{10} = 0.106$$

***Example 5**.
a)
p=0.003
n=1000
np=1000(0.003)=3<5 , so we can use the approximation

$\lambda = np = 3$
$P(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$
$P(x=0) = \dfrac{e^{-3}3^0}{0!} = 0.0498$

b) $\Pr(x \leq 2) = \Pr(x=0) + \Pr(x=1) + \Pr\,(x=2)$

$=\frac{e^{-3}3^0}{0!} + \frac{e^{-3}3^1}{1!} + \frac{e^{-3}3^2}{2!}$
$= e^{-3}\left(1 + 3 + \dfrac{3^2}{2}\right) = e^{-3}(8.5) = 0.423$

N1.
$\lambda = 2$
x=4
$P(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

Pr(x=4)=$\frac{e^{-2}(2)^4}{4!}$=0.09

N2.
$\lambda = 5$
$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$
3 lions Pr(X=3)= $\frac{e^{-5}5^3}{3!} = 0.14$

N3.
$a)\ \lambda = 4.7/hour$
$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Pr(X=3)= $\frac{e^{-4.7}(4.7)^3}{3!} = 0.157$

b) Pr(at least 1)= 1 - Pr(X=0)
= $1 - \frac{e^{-4.7}(4.7)^0}{0!} = 0.99$

N4.
$\lambda = \frac{50}{250} = 0.20$ misprints/per page

$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Pr(X=0)= $\frac{e^{-0.2}(0.2)^0}{0!} = e^{-0.2}$=0.819
N5. $p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$\lambda = 0.25\ per\ sq\ ft = 0.25 \times 12 = 3$ flaws per 12 sq ft
$\lambda = 3$
a) $\Pr(x \le 3) = \Pr(x = 0) + \Pr(x = 1) + \Pr(x = 2) + \Pr\ (x = 3)$
$= \frac{e^{-3}3^0}{0!} + \frac{e^{-3}3^1}{1!} + \frac{e^{-3}3^2}{2!} + \frac{e^{-3}3^3}{3!}$

b) n=10 binomial
no surface flaws...x=0
Pr(x=0)= $\frac{e^{-3}3^0}{0!} = 0.0498$ (Poisson no surface flaws)

Pr(x=3)= $\binom{10}{3}(0.0498)^3(1 - 0.0498)^7 = 0.0104$ (Binomial)

N6.
$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$
$\lambda = 1.8$
X=4

P(x=4)=$\frac{e^{-1.8}(1.8)^4}{4!} = 0.072$

$N7. \lambda = 10\ per\ hour$
$\lambda = 5\ in\ 30\ min$
x= at least 1
$P(at\ least\ 1) = 1 - \Pr(x = 0) = 1 - \frac{e^{-\lambda}\lambda^x}{x!}$=1- $\frac{e^{-5}(5)^0}{0!}$
$= 1 - e^{-5} = 0.993$

N8. $\lambda = 0.3/hour$
This is a binomial question where they want the probability of at least 10 out of 12 intervals have NO log jams. The Pr(X=0) log jams is a Poisson probability calculation
The Poisson formula is Pr(X)=$\frac{e^{-\lambda}\lambda^x}{x!}$
Pr(X=0)=$\frac{e^{-0.3}0.3^0}{0!} = 0.7408 = p$
This is our probability of success in our Binomial question, p
q=1-p=1-0.7408=0.2592

n=12 for the Binomial since we are looking at a total of 12 one hour intervals

We want Pr(Y$\geq 10) = P(Y = 10) + \Pr(Y = 11) + \Pr\ (Y = 12)$
=$\binom{12}{10}(0.7408)^{10}(0.2592)^2 + \binom{12}{11}(0.7408)^{11}(0.2592)^1 + \binom{12}{12}(0.7408)^{12}(0.2592)^0$
$= 0.220710841 + 0.11469037 + 0.027315659 = 0.363$

O. Hyper Geometric Distribution

Example 1.

$\frac{n}{N} = \frac{5}{50} = 0.1 > 0.05 \quad \therefore hypergeometric \qquad N = 50 \quad A = 8$

$n = 5 \quad x = 2 \quad N - A = 50 - 8 = 42$

$n - x = 5 - 2 = 3$

$$P(x) = \frac{\binom{A}{x}\binom{N-A}{n-x}}{\binom{N}{n}} = \frac{\binom{8}{2}\binom{42}{3}}{\binom{50}{5}} = 0.1517$$

Example 2.

$N = 20 \quad A = 4\ defects$

$\frac{n}{N} = \frac{5}{20} = 0.25 > 0.05 \quad \therefore hypergeometric$

$n = 5 \quad x = 0\ defects \qquad N - A = 20 - 4 = 16$

$n - x = 5 - 0 = 5$

$$P(x) = \frac{\binom{A}{x}\binom{N-A}{n-x}}{\binom{N}{n}} = \frac{\binom{4}{0}\binom{16}{5}}{\binom{20}{5}} = 0.2817$$

Example 3.

p=0.025
n=25

a) $\frac{25}{1000} = 0.025 < 0.05, so\ use\ binomial$

Pr(X=3)= $\binom{25}{3}(0.025)^3(0.975)^{22}$=0.0206

b) $\frac{25}{200} = 0.125 > 0.05, so\ use\ hypergeometric$

$$p(x) = \frac{\binom{A}{x}\binom{N-A}{n-x}}{\binom{N}{n}}$$

N=200
n=25
A=0.025x200=5
x=3
N-A=200-5=195
n-x=25-3=22

Pr(X=3)= $\frac{\binom{5}{3}\binom{195}{22}}{\binom{200}{25}}$=0.0138

O1. n/N=5/52=0.096>0.05 so it is hypergeometric

N=52
A=26 red cards
N-A =26 failures (not red)
x=2 successes (red)
n=5 trials

$$p(x) = \frac{\binom{A}{x}\binom{N-A}{n-x}}{\binom{N}{n}}$$

Pr(x=2 red)=$\frac{\binom{26}{2}\binom{52-26}{5-2}}{\binom{52}{5}} = \frac{\binom{26}{2}\binom{26}{3}}{\binom{52}{5}}$

O2. n/N=5/52=0.096>0.05 so it is hypergeometric

N=52
A=13 diamonds
N-A =39 failures (not diamonds)
x=0,1,or2 successes (diamonds)
n=5 trials

Pr(X=0) + Pr(X=1) + Pr(X=2)

$= \frac{\binom{13}{0}\binom{39}{5}}{\binom{52}{5}} + \frac{\binom{13}{1}\binom{39}{4}}{\binom{52}{5}} + \frac{\binom{13}{2}\binom{39}{3}}{\binom{52}{5}}$=0.907

O3. with replacement
binomial p=0.10 defective
q=0.90
n=10

$a)\ \Pr(X \leq 2) = \Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2)$

$= \binom{10}{0}(0.1)^0(0.9)^{10} + \binom{10}{1}(0.1)^1(0.9)^9 + \binom{10}{2}(0.1)^2(0.9)^8$

b) no replacement, so it is hypergeometric

N=20
n=10
A=2 defective
X=0,1,2defective
N-A=20-2=18
n-x=10-0=10, 10-1 = 9 and 10-2=8

$$p(x) = \frac{\binom{A}{x}\binom{N-A}{n-x}}{\binom{N}{n}}$$

Pr(X=0, 1 or 2 defective)= $\frac{\binom{2}{0}\binom{18}{10}}{\binom{20}{10}} + \frac{\binom{2}{1}\binom{18}{9}}{\binom{20}{10}} + \frac{\binom{2}{2}\binom{18}{8}}{\binom{20}{10}}$ =1...you should realize that since are only 2 defective anyway, you can't get more than 2 defective!!!

O4. p=0.03 and n=20
a) $\frac{20}{2000} = 0.01 < 0.05, \textit{so use binomial}$

Pr(X=3)= $\binom{20}{3}(0.03)^3(0.97)^{17}$=0.0183

b) $\frac{20}{200} = 0.10 > 0.05, \textit{so use hypergeometric}$

$$p(x) = \frac{\binom{A}{x}\binom{N-A}{n-x}}{\binom{N}{n}}$$

N=200
n=20
A=0.03x200=6 defective
x=3 defective
N-A=200-6=194
n-x=20-3= 17
Pr(X=3)= $\frac{\binom{6}{3}\binom{194}{17}}{\binom{200}{20}}$=0.0132

O5. $\Pr(x) = \frac{\binom{A}{x}\binom{N-A}{n-x}}{\binom{N}{n}}$

n/N=7/120=0.058>0.05 so it is hypergeometric

N=120 population
A=28 left
n=7 sample
x=3 or 4 left in sample
N-A=120-28=92
n-x=7-3=4 or 7 - 4 =3
Pr(x=3)+Pr(x=4)=$\frac{\binom{28}{3}\binom{92}{4}}{\binom{120}{7}} + \frac{\binom{28}{4}\binom{92}{3}}{\binom{120}{7}}$=0.1539+0.0432 = 0.1971

O6... are selected without replacement?

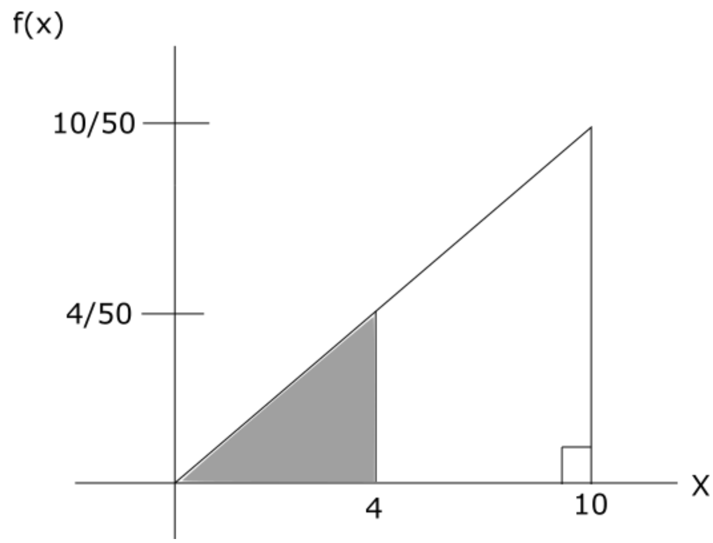
$$p(x) = \frac{\binom{A}{x}\binom{N-A}{n-x}}{\binom{N}{n}}$$

n/N=10/50=0.20>0.05 so it is hypergeometric

N=50 population
A=15 green successes
N – A =50 – 15 = 35 failures
n=10 sample
n-x=10 – 3 = 7
x=3 successes Pr(X=3)=$\frac{\binom{15}{3}\binom{35}{7}}{\binom{50}{10}} = 0.298$

P. Continuous Random Variables

Example 1.
Pr(8<x<15)= Area rectangle = (length)(width)==(15-8)(1/10)=(7)(1/10)=0.70



Example 2. Use the graph to find Pr(X<4).

PrX<4)= area of triangle=$\frac{bh}{2} = \frac{(4)(\frac{4}{50})}{2} = 0.16$

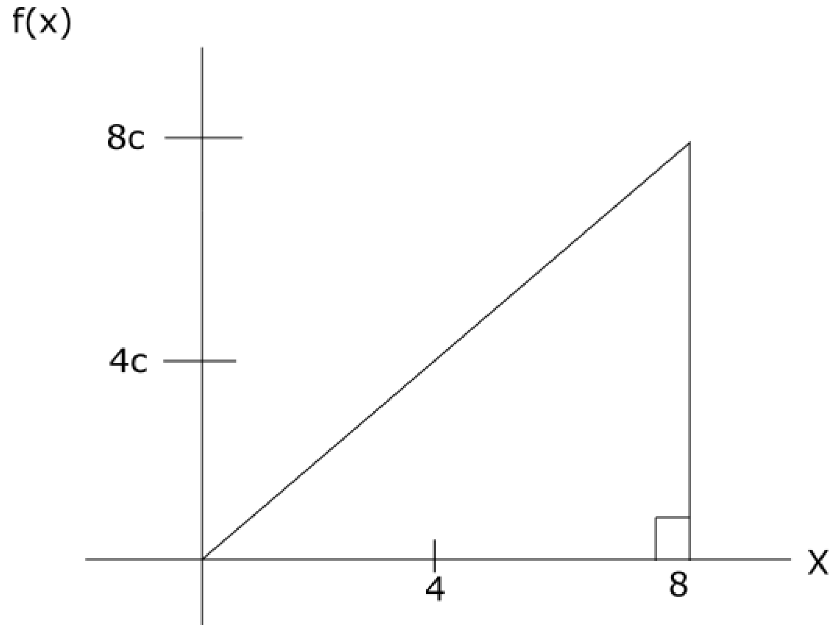
Example 3
if x=1, p(x)=k/1 and if x=2, p(x)=k/2, if x=3, p(x)=k/3...all prob total to 1, so make an equation
$\frac{k}{1} + \frac{k}{2} + \frac{k}{3} = 1$
$\frac{6k}{6} + \frac{3k}{6} + \frac{2k}{6} = 1$ common denominator
$\frac{11k}{6} = 1$ cross multiply and solve k=6/11

Example 4.

Total Area=1
$\frac{bh}{2} = 1$

$\frac{(8)(8c)}{2} = 1$

64c=2
c=2/64=1/32



Example 5.

From the prep book, for uniform distributions,

$\mu_x = \frac{a+b}{2}$ and $\sigma_x = \frac{b-a}{\sqrt{12}}$

and we know the mean squared and the standard deviation are equal in this question and

From (0,b) , we know that a=0.

$$\left(\frac{b}{2}\right)^2 = \frac{b}{\sqrt{12}}$$

$$\frac{b^2}{4} = \frac{b}{\sqrt{12}}$$

$$\sqrt{12}b^2 - 4b = 0$$

$$b\,(\sqrt{12}b - 4) = 0$$

$$\sqrt{12}b - 4 = 0$$

$$b = \frac{4}{\sqrt{12}} = \frac{4}{\sqrt{4}\sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$$

Note: b is not 0 as then it would just be the point (0,0) and not a rectangle.

Example 6.

Since it is uniform between 7:45am and 8:25 am, this is a length along the bottom of 40 minutes, so the width must be 1/40 so that when we multiply lengthxwidth, we get an area of 1.

The probability they report between 8:00 and 8:15am is 1/40(15)=15/40=0.375

Example 7.

To find the value of a, find the area of each of the shapes and set it equal to 1, since the total area is the same as the total probability

A=$\frac{(2)\left(\frac{1}{2}a\right)}{2} + (6)\frac{1}{2}a = 1$

$$1 = \frac{1}{2}a + 3a$$

$$1 = \frac{7}{2}a$$

$$a = \frac{2}{7}$$

Example 8.

uniform between (1.4, 1.6)

a) E(X)=$\frac{a+b}{2} = \frac{1.4+1.6}{2} = 1.5$

If you draw a rectangle with width on the x-axis from 1.4to 1.6...this is =0.2 units...so the length of the rectangle is 1/0.2=5

b) Expected number with voltage more than 1.5volts

=Pr(more than 1.5 volts) x 120 batteries= lengthxwidthx120

=(1.6-1.5)(5)(120) =60 batteries

P1. Pr(3<X<6)=LxW=(3)(1/8)= 3/8

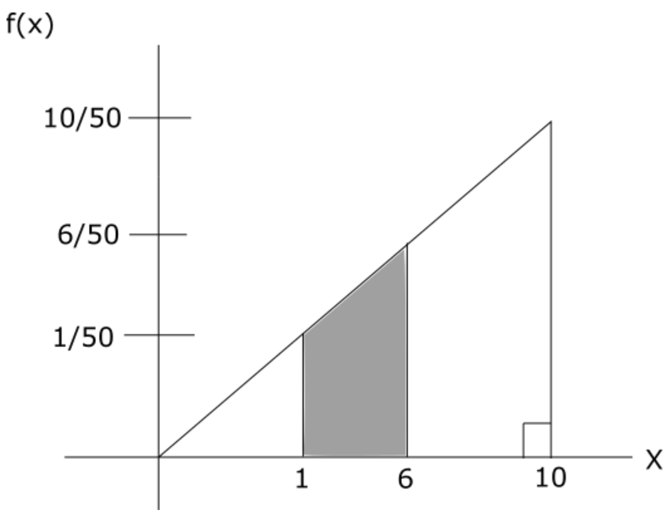
P2.Pr(1<X<6)= Area of Triangle + Area of rectangle

Or Area of triangle between 0 and 6 - Area of triangle from 0 to 1

=(6)(6/50)/2 - (1)(1/50)/2

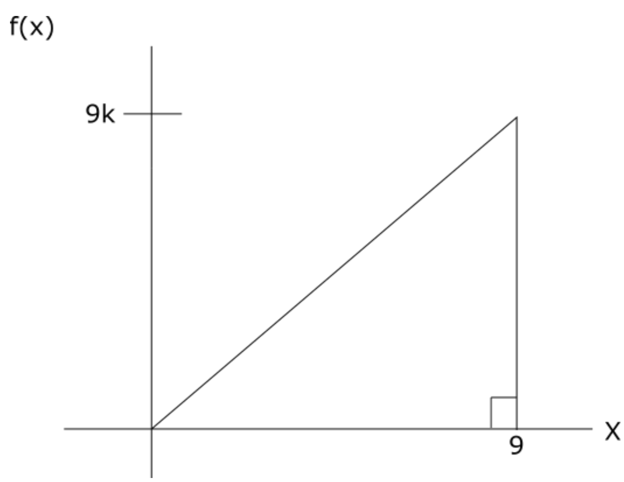
=36/100 - 1/100

=0.35

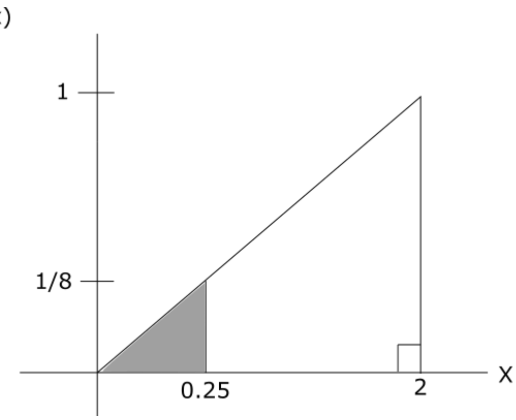


*P3. The total area under the graph must be equal to 1.

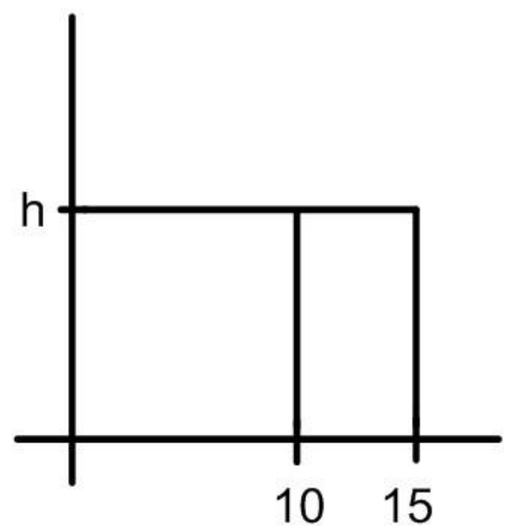
bh/2 = 1
(9)(9k)/2 = 1
81k/2=1
k=2/81



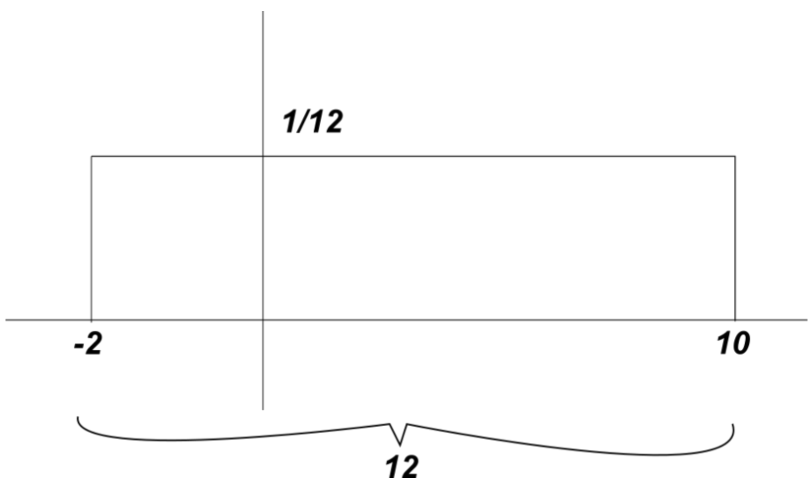
P4. Pr(X<0.25)= area of a triangle= bh/2= (1/4)(1/8)/2= 1/32(1/2) = 1/64



P5.
Area=1
(15)(h)=1
h=1/15
Pr(wait at least 10 min)= lengthxwidth
==(15-10)(1/15)=5(1/15)=1/3



P6.



$$0.25 \times 12 = 3 \quad from - 2 \quad \therefore -2 + 3 = 1$$

The answer is D).

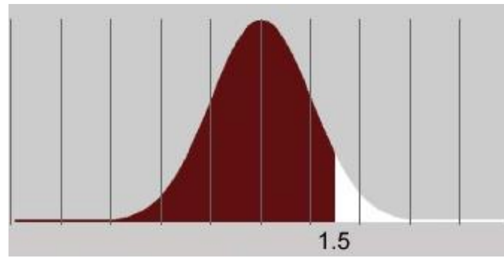
P7. Area of a rectangle is length x width

$$\Pr(x > 0/x < 4) = \frac{\Pr(x>0 \cap x<4)}{\Pr(x<4)}$$
$$= \frac{4\times\frac{1}{12}}{6\times\frac{1}{12}}$$
$$= \frac{4}{6} = \frac{2}{3}$$

Q. Normal Distribution

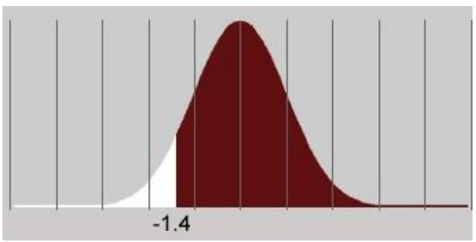
<u>Example 1</u>. Find each of the following probabilities by using the table for Z.

a) Pr[Z<1.5]

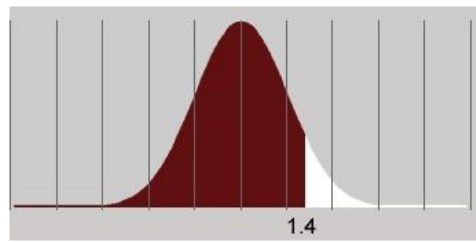


Therefore, the answer is Pr[Z<1.5]= 0.9332

b) Pr[Z>-1.4]

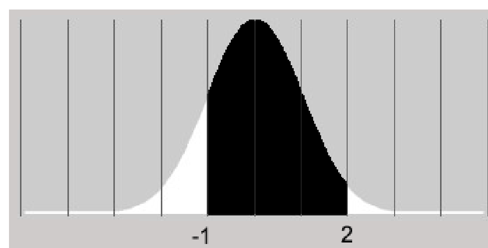


same area as



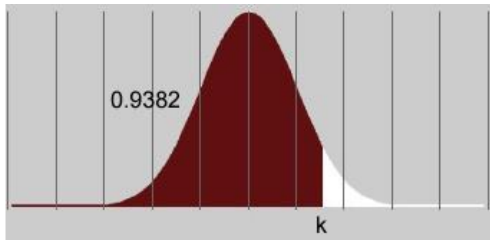
Therefore, the answer is Pr[Z>-1.4] = Pr[Z< 1.4] = 0.9192

c) Pr[-1<Z<2]



Pr[Z<2] - Pr[Z<-1] = 0.9772 - 0.1587 = 0.8185

Example 2.



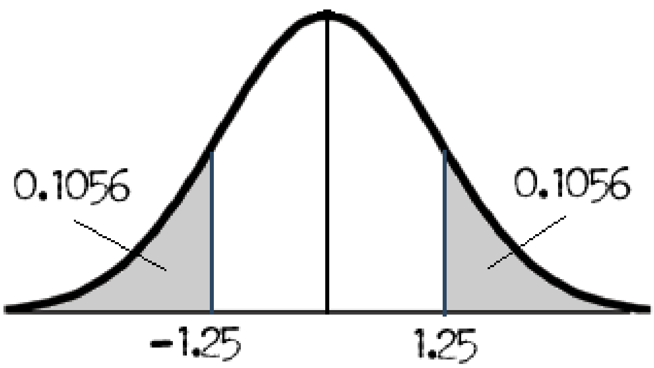
Look up the area 0.9382 and find "k" along the left side of the table.

Therefore, k=1.54

Example 3. Pr(Z<k)=1-0.7=0.3
Look up the area 0.3 and you get k= - 0.525

Example 4.

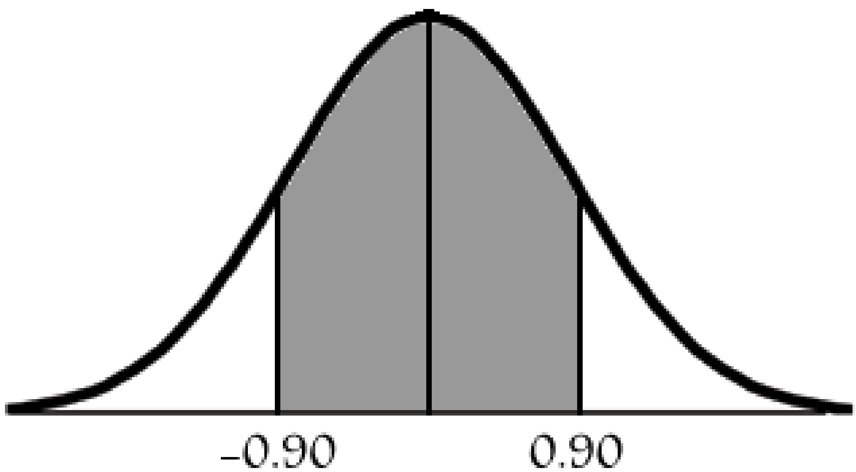
$\Pr((|Z| > 1.25) = 2(0.1056) = 0.2112$



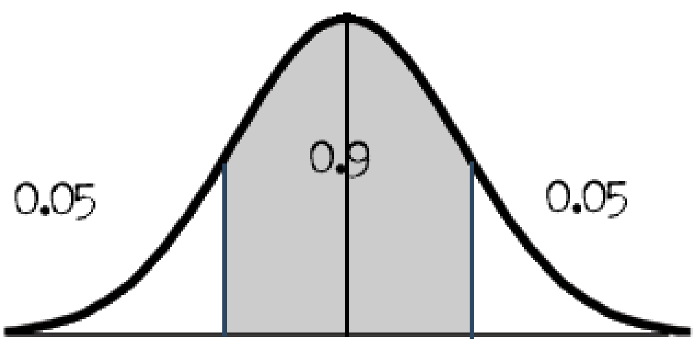
Example 5.

Look up the area Pr(Z<- 0.90)=0.1841 and Pr(Z<0.90)=0.8159 and then subtract them to find the area between them= 0.8159 - 0.1841= 0.6318

Pr(($|Z| < 0.90) = 0.6318$



Example 6.



Look up Area 0.95 in the body (or look up 0.05) and get 1.645, so k=Z=1.645

Q1. Pr (Z<1.6)=0.9452
The answer is b).

Q2. Pr(Z> - 0.80) = Pr(Z<0.80) = 0.7881
The answer is a).

Q3. Pr (-1.20<Z<1.20)= Pr(Z<1.2) - Pr(Z<-1.2)
=0.8849 – 0.1151
=0.7698
The answer is d).

Q4. Pr (Z> 1.76)= 1 - Pr(Z< 1.76)
= 1 - 0.9608
=0.0392
The answer is a).

Q5. If Z is the standard normal random variable, find Pr[-1 < Z < 1].

A. 0.0228	B. 0.9772	C. 0.1587	D. 0.8413	E. None of the above

Pr (-1<Z<1)= Pr(Z<1) - Pr(Z<-1)
=0.8413 – 0.1587
=0.6826
The answer is e).

Q6. Find the value of k if it is known that Pr[k<Z<1.5]= 0.0483 , where Z is the standard normal random variable.

A. 1.2	B. -1.2	C. 0.8849	D. 1.66	E. none of the above

Pr(Z<1.5)= 0.9332
0.9332 – 0.0483=0.8849
Look up area 0.8849 and you get k=1.2

The answer is a).

Q7. Use the table for the standard normal random variable Z to find Pr[-0.65<Z<1.92].

A. 0.6226	B. 0.2284	C. 0.7148	D. 0.2852	E. None of the above

Pr (-0.65<Z<1.92)=Pr(Z<1.92) - Pr(Z<-0.65)
=0.9726 – 0.2578
=0.7148

The answer is c).

Q8. Use the table for the standard normal random variable Z to find a value of k for which Pr[Z<k]= 0.9495

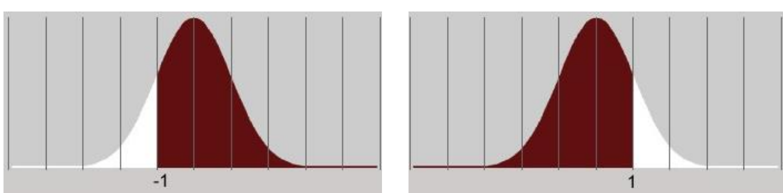
A. 0.9495	B. 0.8264	C. -1.64	D. 1.64	E. None of the above

Pr(Z<k)=0.9495 same area as Pr(Z>k)

Find the area=0.9495 by looking in the body of the chart...we get k=1.64
The answer is d).

Normal Random Variables
Example 1. X is a normal random variable with mean μ=100 and standard deviation σ=10.

Find Pr[X>90].



$$Z = \frac{90 - 100}{10} = -1$$

Pr(X>90)=Pr(Z>-1)= Pr(Z<1)=0.8413

Example 2.
mean= μ=11

$\sigma = 2$

$= \Pr\left(\frac{14-11}{2} < Z < \frac{15-11}{2}\right)$
$= \Pr(1.5 < Z < 2)$
=Pr(Z<2) - Pr(Z<1.5)
=0.9772 -0.9332
=0.044

Example 3. X is a normal random variable with unknown mean μ and standard deviation σ=3. If Pr[X<25]=0.9772, what is the value of μ?

Look up the area 0.9772 in the body and you get Z=2.

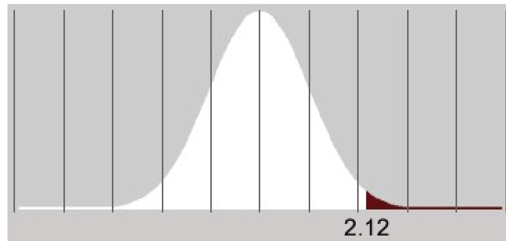
$Z = \frac{X - \mu}{\sigma}$

$2 = \frac{25 - \mu}{3}$

μ =19

Example 4. The IQ's of a large population of children are normally distributed with mean 100.4 and standard deviation 11.6.

(a) What is the probability that a child's IQ is greater than 125?



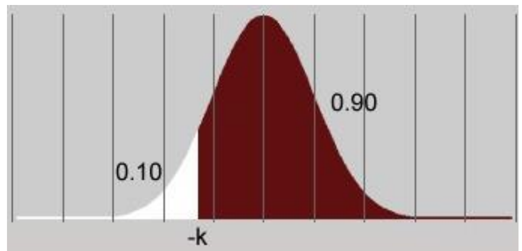
Let X be the child's IQ. Then $X \sim \mathrm{N}(\mu,\sigma)$ with $\mu = 100.4$, $\sigma = 11.6$. So

$$\Pr(X > 125) = \Pr\left(Z = \frac{X-\mu}{\sigma} > \frac{125-100.4}{11.6}\right)$$

$$= \Pr(Z > 2.12) = 1 - \Pr(Z < 2.12) = 1 - 0.9830 = 0.0170$$

(b) About 90% of the children have IQ's greater than what value?

Solution:



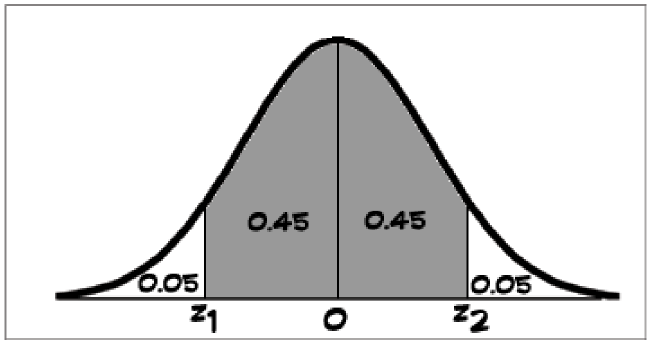
$\Pr(Z > z) = 0.90 \Rightarrow z = -1.28$ (look up the area 0.10 in the body of the table)

$$z = \frac{x-\mu}{\sigma} \Rightarrow x = \mu + z\sigma = \mu - 1.28\sigma$$

So $x = 100.4 - 1.28 \cdot (11.6) = 85.6$.

Thus, 90% of the children have IQ's greater than 85.6.

***Example 5**.



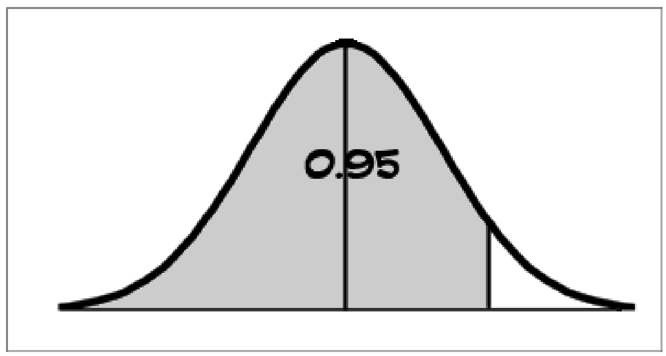
$\mu = 65$
$\sigma^2 = 16$
$\sigma = 4$
$Z = \dfrac{X-\mu}{\sigma}$

Look up area 0.95 to find Z_2=1.645

Look up area 0.05 to find Z_1= -1.645

$-1.645 = \dfrac{X-65}{4}$
X_1=58.4

$1.645 = \frac{X-65}{4}$
X_2=71.6



The interval is from (58.4, 71.6)

Example 6.

$\mu = 19.5oz$
$\sigma = 0.6$

Pr(X<c)=0.70 find c

Area=0.70 then Z=0.525

$$0.525 = \frac{X - 19.5}{0.6}$$

X=c=19.82 oz

Example 7. $n = 25$

$x = 3\ lost\ money\ (is\ Binomial\ with\ p = prob\ x < 0$ (normal))

$p = prob\ lost\ money = \Pr\ (x < 0)$

$z = \frac{x-\mu}{\sigma}$

$\Pr(x < 0) = \Pr\ (z < \frac{0-9.2}{5.1})$

$= \Pr\ (z < -1.8)$

$= 0.0359$

$p = 0.0359$

$q = 1 - 0.0359 = 0.9641$

$\therefore \Pr(x = 3) = \binom{25}{3}(0.0359)^3(0.9641)^{22}$

$= 0.0475 \quad or \quad 4.75\%$

Q9.... $\Pr(X<520)=\Pr(Z<\frac{520-500}{20})=\Pr(Z<1)= 0.8413$

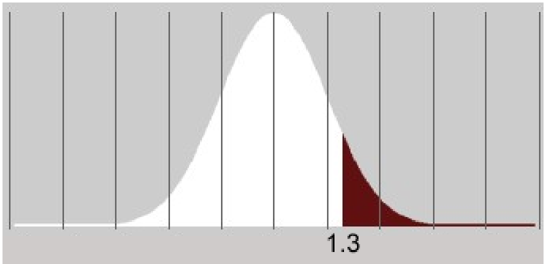
Q10.
X is a normal random variable with mean 35 and standard deviation 5.

$\Pr(30<X<40)=\Pr(\frac{30-35}{5} < Z < \frac{40-35}{5}) = \Pr(-1 < Z < 1)$

$=\Pr(Z<1) - \Pr(Z<-1)$

$=0.8413 - (1-0.8413)$
$=0.8413 - 0.1587$
$=0.6826$

Q11. ...who took the test had a score greater than 630?



Let X be the test score. Then $X \sim N(\mu,\sigma)$ with $\mu = 500$, $\sigma = 100$. So

$$\Pr(X > 630) = \Pr\left(Z = \frac{X-\mu}{\sigma} > \frac{630-500}{100}\right)$$
$$= \Pr(Z > 1.3) = 1 - \Pr(Z < 1.3) = 1 - 0.9032 = 0.0968.$$

Q12. (a) What percentage of pregnancies last less than 240 days?

Let X be the length of the pregnancy in days. Then $X \sim N(\mu,\sigma)$ with $\mu = 266$, $\sigma = 16$.

So $$\Pr(X < 240) = \Pr\left(Z = \frac{X-\mu}{\sigma} < \frac{240-266}{16}\right) = \Pr(Z < -1.63) = 0.0516.$$

(b) What percentage of pregnancies last between 240 and 270 days?

$$\Pr(240 < X < 270) = \Pr\left(\frac{240-266}{16} < Z < \frac{270-266}{16}\right) = \Pr(-1.63 < Z < 0.25)$$
$$= \Pr(Z < 0.25) - \Pr(Z < -1.63) = 0.5987 - 0.0516 = 0.5471.$$

(c) How long do the longest 20% of pregnancies last?

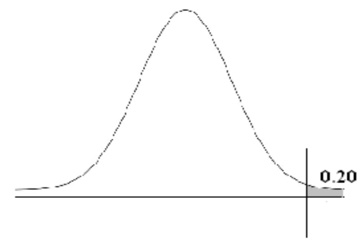
Look up the area=0.80 in the body and get Z=0.84

$\Pr(Z > z) = 0.20 \Rightarrow z = 0.84.$

$$z = \frac{x-\mu}{\sigma} \Rightarrow x = \mu + z\sigma = \mu + 0.84\sigma$$

$$= 266 + 0.84 \cdot (16) = 279.44.$$

Therefore, the longest 20% of pregnancies last more than 279 days.



Q13. (a) What is the probability of getting a 91 or less on the exam?

Let X be the final grade. Then $X \sim N(\mu, \sigma)$ with $\mu = 73$, $\sigma = 8$. Then

$$\Pr(X \le 91) = \Pr\left(Z = \frac{X-\mu}{\sigma} \le \frac{91-73}{8}\right) = \Pr(Z < 2.25) = 0.9878.$$

(b) What percentage of students scored between 65 and 89?

$$\Pr(65 < X < 89) = \Pr\left(\frac{65-73}{8} < Z < \frac{89-73}{8}\right) = \Pr(-1 < Z < 2)$$

$$= \Pr(Z < 2) - \Pr(Z < -1) = 0.9772 - 0.1587 = 0.8185.$$

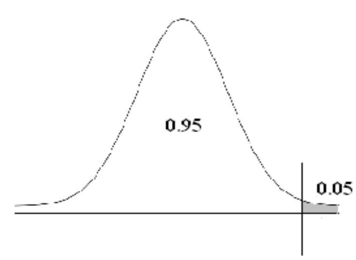
c) Only 5% of the students taking the test scored higher than what grade? Look up 0.95 in the body and get 1.645

$\Pr(Z > z) = 0.05 \Rightarrow z = 1.645.$

$$z = \frac{x-\mu}{\sigma} \Rightarrow x = \mu + z\sigma = \mu + 1.645\sigma$$

So $x = 73 + 1.645 \cdot (8) = 86.16.$

Therefore, 5% of the students scored higher than 86%.



Q14. (a) Find the probability that the monkey's weight is less than 13 pounds.

Let X be the rhesus monkey's weight in pounds. Then $X \sim N(\mu, \sigma)$ with $\mu = 15$, $\sigma = 3$.

So $$\Pr(X < 13) = \Pr\left(Z = \frac{X-\mu}{\sigma} < \frac{13-15}{3}\right) = \Pr(Z < -0.67) = 0.2514.$$

(b) Find the probability that the weight is between 13 and 17 pounds.

Solution:

$$\Pr(13 < X < 17) = \Pr\left(\frac{13-15}{3} < Z < \frac{17-15}{3}\right) = \Pr(-0.67 < Z < 0.67)$$
$$= \Pr(Z < 0.67) - \Pr(Z < -0.67) = 0.7486 - 0.2514 = 0.4972.$$

(c) Find the probability that the monkey's weight is more than 17 pounds.

$$\Pr(X > 17) = \Pr\left(Z > \frac{17-15}{3}\right) = \Pr(Z > 0.67) = 1 - \Pr(Z < 0.67)$$
$$= 1 - 0.7486 = 0.2514.$$

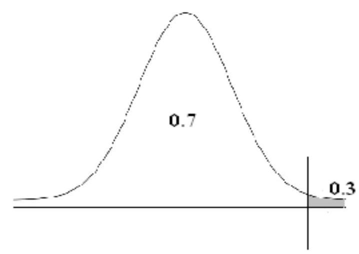
Q15. **(a)** What is the shortest time spent waiting for a heart transplant that would still place a patient in the top 30% of waiting times?

Let X be the waiting time (in days). Then

$X \sim N(\mu, \sigma)$ with $\mu = 127$, $\sigma = 23.5$.

So $\Pr(Z > z) = 0.30 \Rightarrow z = 0.52$. (look up area 0.70 in the body and get Z=0.52)

$$z = \frac{x-\mu}{\sigma} \Rightarrow x = \mu + z\sigma = \mu + 0.52\sigma$$
$$= 127 + 0.52 \cdot (23.5) = 139.2.$$

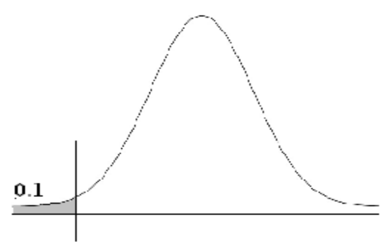


Therefore, 30% of the patients must wait for more than 139 days for a heart transplant.

(b) What is the longest time spent waiting for a heart transplant that would still place a patient in the bottom 10% of waiting times?

$\Pr(Z < z) = 0.10 \Rightarrow z = -1.28$. (look up area 0.10 in the body and get Z= -1.28)

$$z = \frac{x-\mu}{\sigma} \Rightarrow x = \mu + z\sigma = \mu - 1.28\sigma$$
$$= 127 - 1.28 \cdot (23.5) = 96.9.$$



Therefore, 10% of the patients have to wait for less than 97 days for a heart transplant.

Best of luck on the exam!!!!!

Test 2 Material Covered in Fall 2024

A. The Exponential Distribution

Example 1. Jobs are sent to a printer at an average of 3 jobs per hour.

y= # of jobs sent to the printer per hour   y=0,1,2,...
$y \sim Poisson\ (\lambda = 3 = \mu_y)$

x=time between jobs at the printer in hours   x>0
$x \sim Exponential\ (\lambda = 3)$
E(x)=$\frac{1}{3}$ *which means a job arrives every* $\frac{1}{3}$ *of an hour*

a) What is the expected time between jobs?

Solution:

$\lambda = 3$
expected time=E(x)= $\frac{1}{\lambda} = \frac{1}{3}$ hour

b) What is the probability that the next job is sent within 10 minutes?
$\lambda = 3$ per hour

10 min=0.16666 an hour

$\Pr(x \le a) = 1 - e^{-\lambda a}$

$\Pr(x \le 10min) = \Pr\ (x \le 0.16666)$hr

$= 1 - e^{-\lambda a}$
$= 1 - e^{-3(0.166666)}$
=0.393

Example 2.
$\mu_x = 100\ 000$ miles

y= # of failures (miles) y=0,1,2,...
$y \sim Poisson\ (\lambda = 100\ 000 = \mu_y)$

x=distance between failures (in # miles) x>0
$x \sim Exponential\ (\lambda = 100\ 000)$

E(x)=100 000 miles
$which\ means\ a\ car\ travels\ 100\ 000\ miles\ on\ average\ before\ failure$

$$\mu_x = \frac{1}{\lambda}$$
$$100\ 000 = \frac{1}{\lambda}$$

$$\lambda = \frac{1}{100\ 000} = 0.00001$$

$$\Pr(x \leq 70\ 000) = 1 - e^{-\lambda b}$$

$$= 1 - e^{-0.00001(70\ 000)}$$
$$= 1 - e^{-0.7}$$
$$= 1 - 0.497$$
=0.503

Example 4.
$\lambda = 3$/hour

y= # of students arriving per hour y=0,1,2,...
$y \sim Poisson\ (\lambda = 3 = \mu_y)$

x=time between students x>0
$x \sim Exponential\ (\lambda = 3)$
E(x)=$\frac{1}{3}$ $which\ means\ a\ student\ arrives\ every\ \frac{1}{3} of\ an\ hour$

a) Prob. more than 10 min before next person arrives

Exponential

Pr(X>10min)=Pr(X>10/60hr)=Pr(X>0.16667)

Pr(X>c)= $e^{-\lambda c}$
Pr(X>0.16667)=$e^{-3(0.1667)} = 0.6065$

b) Prob time between arrivals is between 10 and 20 min?
Exponential Pr(a<X<b)= $e^{-\lambda a} - e^{-\lambda b}$

Pr(10<X<20min)=Pr(10/60<X<20/60 hr)= Pr(0.166667<X<0.3333333)

=$e^{-3(0.1666667)} - e^{-3(0.333333)} = 0.6065 - 0.3679 = 0.239$

Example 5.
$\lambda = 0.6\ downs/hour$

a) Pr(X>c)= $e^{-\lambda c}$
Pr(X>3)= $e^{-0.6(3)} = 0.165$

b) Pr(X$\geq 4/X > 1)$ =Pr(X$\geq$ 3)= $e^{-0.6(3)} = 0.165$

Example 6. $\lambda = 16$/hour

y= # of calls arriving per hour y=0,1,2,...
$y \sim Poisson\ (\lambda = 16 = \mu_y)$

x=time between calls x>0
$x \sim Exponential\ (\lambda = 16)$
E(x)=$\frac{1}{16}$ $which\ means\ a\ call\ arrives\ every\ \frac{1}{16} of\ an\ hour$

Pr ($X < 15/$X>10) is a conditional probability...BUT, remember the MEMORYLESS property of the exponential function (it doesn't remember you have waited 10 minutes. But, it doesn't say Pr(X>15) so it is just finding the probability that the next call occurs in the next 5 minutes, or less than/equal to 5)

Pr$(X \leq a) = 1 - e^{-\lambda a}$
=Pr(X<5min)=Pr(X<5/60 hr))=Pr(X<0.08333)
is in the next 5 min=$1 - e^{-16(0.08333)} = 1 - 0.264 = 0.736$

NOTE: If it just said what is the probability the next call arrives between 10 and 15 min from now....it would be:

Pr(10<X<15min)=Pr(1/6<X<1/4 hr)= $e^{-\lambda a} - e^{-\lambda b}$

= $e^{-16\left(\frac{1}{6}\right)} - e^{-16\left(\frac{1}{4}\right)}$=0.0695-0.0183=0.0512

Example 7.

$$\begin{aligned}
\Pr(x < t) &= 0.75 \\
1 - e^{-\lambda a} &= 0.75 \\
1 - e^{-16a} &= 0.75 \\
0.25 &= e^{-16a} \\
\ln 0.25 &= \ln e^{-16a} \\
\ln 0.25 &= -16a \ln e \\
-1.38629 &= -16a \\
a &= 0.08664\ hour\ \times 60 = 5.2\ minutes.
\end{aligned}$$

Example 8.

$\mu = 0.25 \ \ \lambda = \frac{1}{\mu} = \frac{1}{0.25} = 4$

$Pr(x < c) = 0.50$

$1 - e^{-\lambda c}$=0.50

$1 - e^{-4c} = 0.50$

$0.5 = e^{-4c}$

$ln0.5 = lne^{-4c}$

$ln0.5 = -4clne$

$ln0.5 = -4c$

$c = 0.17 \ hours \ or \ 0.17 \times 60 = 10.4 \ minutes$

Practice Exam Questions on Exponential Distributions

A1. The time in hours required to repair a machine is an exponential random variable with $\lambda = 1/2$.

y= # of repairs per hour y=0,1,2,...
$y \sim Poisson\ (\lambda = \frac{1}{2} = \mu_y)$

x=time between repairs x>0
$x \sim Exponential\ (\lambda = 1/2)$
E(x)=$\frac{1}{2}$ $which\ means\ a\ student\ arrives\ every\ \frac{1}{2} of\ an\ hour$
a) What is the probability that the repair time exceeds 3 hours?

$\lambda = \frac{1}{2} = 0.5$
$\mu_x = \frac{1}{\lambda} = 2$

Pr(X>c)= $e^{-\lambda c}$

Pr($x > 3$)=$e^{-0.5(3)} = e^{-1.5}$=0.223

b) What is the conditional probability that a repair takes at least 10 hours, given that its duration exceeds 8 hours?

Pr(X≥10/X>8)= $\Pr(X \geq 2)\ by\ memoryless\ property$
$= e^{-0.5(2)} = 0.368$

A2. On the average, a certain computer part lasts 12 years. The length of time the computer part lasts is exponentially distributed.

y= # of computer repairs y=0,1,2,...
$y \sim Poisson\ (\lambda = \frac{1}{12} = \mu_y)$

x=# of years before repair is needed x>0
$x \sim Exponential\ (\lambda = \frac{1}{12})$
E(x)=12 $yr\ = \mu\ which\ means\ a\ student\ arrives\ every\ \frac{1}{12} of\ an\ hr$

What is the probability that a computer part lasts more than 6 years?

$\mu = 12$ years
$\lambda = 1/12$ yr
Pr(X>6)=$e^{-\frac{1}{12}(6)} = e^{-0.5} = 0.607$

A3.
Let x be the time between accidents
X$\sim Exponential\ with\ \lambda = 3$

Pr(X>2)=$e^{-3(2)} = e^{-6}$=0.002479

A4. Let x be the time between customers
X$\sim Exponential\ with\ \mu = 10$
$\lambda = 1/10$=0.1

a) Pr(X>20)= $e^{-0.1(20)} = 0.135$

b) Pr(X≥15/X>12)= Pr$(X \geq 3)\ by\ Memoryless\ property = e^{-0.1(3)}$=0.7408

*A5. $mean = 25, so\ \lambda = 1/25$

a) Pr(X>35)= $e^{-\frac{1}{25}(35)} = 0.247$

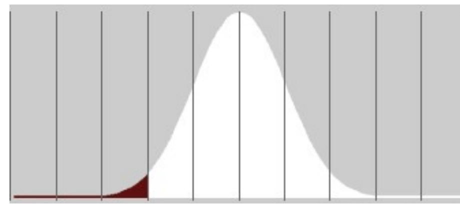
b)Pr(spend more than 30 minutes/ still in store after 25 minutes)
=Pr(X>5 minutes)
=$e^{-1/25(5)} = 0.819$

B. Normal Approximation to the Binomial Distribution

Example 1.

Pr(X=4) use approximation Pr(3.5<Y<4.5)

Pr(X<5)=Pr(X$\leq$ 4) $= \Pr(Y < 4.5)$



Pr(X$\leq$ 7) $= \Pr(Y < 7.5)$

Pr(X>8)=Pr(X$\geq$ 9) $= \Pr(Y > 8.5)$

Example 2.

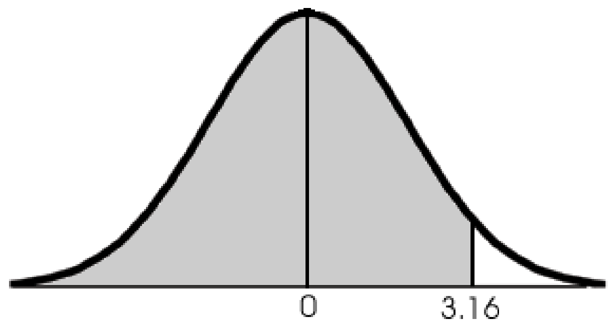
$np = 200(0.1) \geq 5$ √

$nq = 200(0.9) \geq 5$ √

$\therefore$ *use normal approx*

$\mu = np = 200(0.1) = 20$

$\sigma = \sqrt{npq} = \sqrt{20(0.9)} = 4.24$



$\Pr(x \geq 35) = \Pr(y > 34.5)$

$= \Pr\left(Z > \frac{34.5-20}{4.24}\right)$

$= \Pr(Z > 3.42)$

$= 1 - 0.9997$

$= 0.0003$

Example 3.

$np = 100\left(\frac{1}{6}\right) \geq 5$ √

$nq = 100\left(\frac{5}{6}\right) \geq 5$ √

$\therefore$use normal approx.

$\mu = np = 100\left(\frac{1}{6}\right) = 16.\overline{6}$

$\sigma = \sqrt{npq} = \sqrt{16.6(\frac{5}{6})} = 3.73$

$n = 100$

$\Pr(23.5 < y < 24.5)$

$= Pr\left(\frac{23.5-16.\overline{6}}{3.73} < Z < \frac{24.5-16.\overline{6}}{3.73}\right)$

$= \Pr(1.83 < Z < 2.10)$

$= \Pr(Z < 2.10) - \Pr(Z < 1.83)$

$= 0..9821 - 0.9664$

$= 0.0157$

Example 4.

$\Pr(9.5 < y < 15.5)$

$= \Pr\left(\frac{9.5-16.\overline{6}}{3.73} < Z < \frac{15.5-16.\overline{6}}{3.73}\right)$

$= \Pr(-1.92 < Z < -0.31)$

$= \Pr(Z < -0.31) - \Pr(Z < -1.92)$

=0.3783-0.0274

$= 0.3509$

Example 5.

np=60(0.25)=15
nq=60(0.75)=45
both greater or equal to 5

n=60, p=0.25 and q=0.75
$\mu = np = 60(0.25) = 15 \; and \; \sigma = \sqrt{npq} = \sqrt{60(0.25)(0.75)} = 3.354$
$\Pr(X \geq 18) = \Pr(Y > 17.5)$
$\Pr\left(Z > \frac{17.5 - 15}{3.354}\right)$

=Pr(Z>0.75)

=1- 0.7734
=0.2266

Practice Exam Questions on Approximation to the Normal Distribution

*B1. n=100 check np=50≥ 5 $and\ nq = 50 \geq 5$, so we can approximate

p=0.5

q=0.5

$\mu = np = 100(0.5) = 50$
$\sigma = \sqrt{npq} = \sqrt{50(0.5)} = 5$

Pr(X>65)=Pr(Y>65.5)=Pr$\left(Z > \frac{65.5-50}{5}\right) = \Pr(Z > 3.1) = 1 - \Pr(Z < 3.1) =$
$1 - 0.999 = 0.001$

*B2. $\mu = 100, \sigma = 10$

Pr(X$\leq$ 115) $= \Pr(Y < 115.5) = \Pr\left(Z < \frac{115.5-100}{10}\right) = \Pr(Z < 1.55) = 0.9394$

*B3. a) check np=30≥ 5 $and\ nq = 180(\frac{5}{6}) \geq 5$, so we can approximate

$\mu = np = 180(1/6) = 30$

b) $\sigma = \sqrt{npq} = \sqrt{30(5/6)} = 5$

c) $\Pr(X \leq 15) = \Pr(Y < 15.5) = \Pr\left(Z < \frac{15.5-30}{5}\right) = \Pr(Z < -14.5/5) = \Pr(Z < -2.9)$
=Pr(Z>2.9)
=1-Pr(Z<2.9)
=1-0.9981=0.0019

*B4.
a) check np=50≥ 5 $and\ nq = 50 \geq 5$, so we can approximate

$\mu = np = 100(0.5) = 50$

b) $\sigma = \sqrt{npq} = \sqrt{50(0.5)} = 5$

c) $\Pr(\text{X} \geq 45) = \Pr(\text{Y} > 44.5) = \Pr\left(\text{Z} > \frac{44.5-50}{5}\right) = \Pr(\text{Z} > -1.1) = 1 - 0.1357 = 0.8643$

B5. $x = \#\, tails$
$n = 18$
$q = heads = \frac{2}{3}$
$p = tails = \frac{1}{3}$ (p must be tails as X=# of tails)

check np=6≥ 5 $and\ nq = 18\left(\frac{2}{3}\right) = 12 \geq 5$, so we can approximate

a) $\mu = np$
$= 18\left(\frac{1}{3}\right) = 6$

b)

$\sigma = \sqrt{npq}$
$= \sqrt{6\left(\frac{2}{3}\right)} = \sqrt{4} = 2$

c)
$Z = \frac{Y - \mu}{\sigma} = \frac{12.5 - 6}{2} = 3.25$

$Pr(X \leq 12)$
$= Pr(Y < 12.5)$
$= Pr(Z < 3.25)$
$= 0.9994$

B6.

$p = get\ red = 0.5$
$q = 0.5$

check np=50$\geq$ 5 $and\ nq = 50 \geq 5$, so we can approximate

a) $= np$
$= 100(0.5) = 50$

b) $= \sqrt{npq}$
$= \sqrt{50(0.5)} = 5$

c)
$Z_1 = \frac{41.5 - 50}{5} = -1.7$
$Z_2 = \frac{56.5-50}{5}$ =1.3

$Pr(42 \leq X \leq 56)$
$= Pr(41.5 < Y < 56.5)$
$= Pr(-1.7 < Z < 1.3)$
$= 0.9032 - 0.0446$

=0.8586

B7. $n = 100$
$p = q = 0.5$
check np=50$\geq$ 5 $and\ nq = 50 \geq 5$, so we can approximate

$\mu = np = 50$
$\sigma = \sqrt{npq} = \sqrt{25} = 5$

$Pr(X \leq 55)$
$= Pr(Y < 55.5)$

$$Z = \frac{Y - \mu}{\sigma} = \frac{55.5 - 50}{5} = \frac{5.5}{5} = 1.1$$

$\therefore Pr(Z < 1.1) = 0.8643$

B8. n=100

p=0.5

q=0.5

$\mu = np = 100(0.5) = 50$
$\sigma = \sqrt{npq} = \sqrt{50(0.5)} = 5$

Pr(X>60)=Pr(Y>60.5)=Pr $\left(Z > \frac{60.5-50}{5}\right) = \Pr(Z > 2.1)$
$= 1 - \Pr(Z < 2.1) = 1 - 0.9821 = 0.0179$

*B9. $\mu = 100$

$\sigma = 10$

Pr(X$\leq 110) = \Pr(Y < 110.5) = \Pr\left(Z < \frac{110.5-100}{10}\right)$

$= \Pr(Z < 1.05) = 0.8531$

*B10. a) Find the expected value of X.

$\mu = np = 100(0.5) = 50$
b) Find the standard deviation of X.

$\mu = np = 100(0.5) = 50$
$\sigma = \sqrt{npq} = \sqrt{50(0.5)} = 5$

c) Find an approximation for the probability that Stacey turns up no more than 50 red cards.

$\Pr(X \leq 50) = \Pr(Y < 50.5) = \Pr\ (Z < \frac{50.5-50}{5}$)=Pr(Z<0.1)=0.5398

*B11.a) Find E(X).

q=2/3 miss
p=1/3 get shot in
n=18

E(X)=np=18(1/3)=6
b) Find $\sigma(X)$.

$\sigma = \sqrt{npq} = \sqrt{6(\frac{2}{3})} = 2$

c) Find the probability of getting between 8 and 10 shots (inclusive) in the net.

$\Pr(8 \leq X \leq 10) = \Pr(7.5 < Y < 10.5) = \Pr\ (\frac{7.5-6}{2} < Z < \frac{10.5-6}{2}$)=Pr(0.75<Z<2.25)
= Pr(Z<2.25) - Pr(Z<0.75)=0.9878 - 0.7734=0.2144

C. Methods of Sampling

Example 1. b) stratified because we are doing a random sample of each group or "strata"

Example 2. c) systematic

Example 3. a) cluster

Random Digits

Example 1. Choose numbers 6,9,0,4 and call Wendy's, Taco Bell, Archies and Jack Astors (order doesn't matter)

Example 2. As soon as there are more than 10 restaurants, we would need to use two digits to label them

01Wendy's
02Tim Hortons
03Swiss Chalet
04Burger King
05Jack Astors
06McDonald's
07Archie's
08East Side Marios
09Montanas
10Taco Bell
11Harveys
12Pizza Hut
13Red Lobster

If we use the same random list of numbers, we need to look for two-digit numbers that APPEAR in our list. A number such as the first two-digit number "69" doesn't apply because we don't have any restaurant labeled 69.
69043 81235 90721 30174 97245

Which restaurants would we select if we wanted a random sample of four restaurants?

Circle two-digit numbers until you get four numbers that are in the list from 01 to 13.

04, 12, 13, 01 are the first four numbers, so we would choose Burger King, Pizza Hut, Red Lobster and Wendy's.

Example 3.

Subjects must all have the same number of digits...they will be 01, 02, ..., 16

Circle two numbers at a time, without skipping any, until you find four people

81**05**7 271**02** 56**02**7 55892 33**06**3 41842 81868 7**10**35 43367

We got 02 twice, but you can't count the same person twice, so we get 05, 02, 06 and 10

Practice Exam Questions

C1. The answer is (c).

C2. The answer is (d).

C3. The answer is (c).

C4. The answer is (d).

C5. The answer is (d).

C6. The answer is (b).

C7. The answer is (e).

C8. The answer is (e).

C9. The answer is (b).

C10. The answer is (c).

C11. The answer is (c).

C12. The answer is (b).

C13. The answer is (c).

C14. The answer is (d).

C15. The answer is (d).

C16. The answer is (a).

C17. The answer is (d).

C18. The answer is (d).

C19. The answer is (a).

C20. The answer is (b).

C21. The answer is (b).

C22. The answer is (a).

C23. The answer is (b).

C24. The answer is (c).

C25. The answer is (c).

C26. The answer is (c).

C27. The answer is (a).

C28.

11793 20495 05907 11384 44982 20751 27498 12009

Circle one number at a time and do so until you obtain three names

1,1,7,9...we would use 1, 7, 9 since we can't pick 1 twice...so, call, Chapman, Stamm and Wright
The answer is c).

C29.
81507 27102 56027 55892 33063 41842 81868 71035 09001

The first four to get the new medication are
8,1,5,7 since 0 doesn't represent a name

Then, we get **2**, 7, 1, 0, 2, 5, **6**, 0, 2, 7, 5, 5, 8, 9, 2, **3,** 3, 0, 6, 3**, 4.**.. The bolded ones are the ones we take since all others are repeats of the first four subjects who are already getting the medication

So, 2, 6, 3, 4 are the subjects to get the placebo...meaning, Chapman, Lovett, Dennis and Fitzgerald...note since there are only 8, we could just assume it was the four people we didn't get at the start, but since there could be 30 people, you need to know the method
The answer is (d).

C30.
A). The explanatory variable is the herbal tea. The answer is (b).

B). The confounding variable isn't a variable being studied, but any that will mess up your study and make the cause and effect difficult to prove. Since the elderly might be doing better from having extra visits and attention, their increased cheerfulness might be due to the company and have nothing to do with the tea.
The answer is (d).

C31. **14 42** 92 60 56 **31 42 48 03** 71 65 10 36 22 53 22 49 06

We would pick two numbers at a time, from left to right, until we get 5 numbers that are between 01 and 50

14, 42, 31, 48,03..since we don't count 42 twice

The answer is (d).

D. Sampling Distributions

Example 1.

$$total \ \ 82\ kg$$

$$\bar{x} = \frac{82}{20} = 4.1$$

$$\Pr(\bar{x} < 4.1) = \Pr\left(Z < \frac{4.1-4.2}{1.05/\sqrt{20}}\right)$$

$$= \Pr(Z < -0.43)$$

$$= 0.3336$$

Example 2.

$$\mu = 10.35 \qquad \sigma = 0.8 \qquad n = 100$$

$$\Pr(\bar{x} > 10.45) = \Pr\left(z > \frac{10.45-10.35}{\frac{0.8}{\sqrt{100}}}\right)$$

$$= \Pr(z > 1.25)$$

$$= 1 - 0.8944 = 0.1056$$

Example 3.

This adjustment is called the FINITE CORRECTION FACTOR

When $\frac{n}{N} > 0.05, the\ following\ equation\ should\ be\ used$:

** Look for questions where the size of the population N is given

Check $\frac{n}{N} = \frac{80}{800} > 0.05$

$$\sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{800-80}{800-1}} = 0.949276781$$

$$\Pr(\bar{x} < 87) = \Pr\left(Z < \frac{87-84}{0.949276781\left(\frac{9}{\sqrt{80}}\right)}\right)$$

$$= \Pr(Z < 3.16) = 0.9992$$

Example 4. The answer is D).

Example 5. The answer is B).

Practice Exam Questions on Sampling Distributions

D1. (a) Let X be the diameter of a ping pong ball. Then $X \sim \text{N}(\mu, \sigma^2)$ with $\mu = 33.0$, $\sigma = 1.0$

$$\Pr(32.5 < X < 33.0) = \Pr\left(\frac{32.5 - 33.0}{1.0} < Z < \frac{33.0 - 33.0}{1.0}\right) = \Pr(-0.5 < Z < 0.0)$$

$$= \Pr(Z < 0.0) - \Pr(Z < -0.5) = 0.5000 - 0.3085 = 0.1915.$$

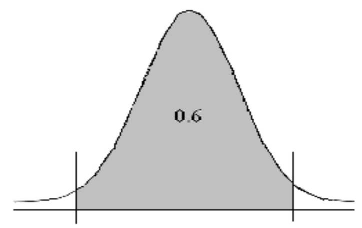
(b) $$\Pr(33.3 < X < 33.8) = \Pr\left(\frac{33.3 - 33.0}{1.0} < Z < \frac{33.8 - 33.0}{1.0}\right) = \Pr(0.3 < Z < 0.8)$$

$$= \Pr(Z < 0.8) - \Pr(Z < 0.3) = 0.7881 - 0.6179 = 0.1702.$$

(c) $\Pr(-z < Z < z) = 0.6 \;\Rightarrow\; 2\Pr(0 < Z < z) = 0.6$

1-0.60=0.40 total on both sides, which means 0.20 on each side

Total area below the line is then 0.80…look up 0.80 in the body



$\Rightarrow \Pr(Z < z) = 0.8.$

$\Rightarrow z = 0.84.$

The two z-scores are $z = \pm 0.84$, so since $x = \mu + z\sigma$, the two diameters are

$$\mu - 0.84\sigma = 33.0 - 0.84 \cdot (1.0) = 32.16$$

and $$\mu + 0.84\sigma = 33.0 + 0.84 \cdot (1.0) = 33.84.$$

That is $\Pr(32.16 < X < 33.84) = 0.6$, so 60% of the ping pong balls will have diameters between 32.16 mm and 33.84 mm.

D2.(a) Let X be the number of minutes using e-mail. Then $X \sim \text{N}(\mu, \sigma)$ with $\mu = 8$, $\sigma = 2$

The sample size is $n = 25$, so $\mu_{\bar{X}} = \mu = 8$ and $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{25}} = 0.40$.

The z-score is given by $Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} < \frac{\bar{X} - 8}{0.40}$. Therefore

$$\Pr(7.8 < \bar{X} < 8.2) = \Pr\left(\frac{7.8 - 8}{0.40} < Z < \frac{8.2 - 8}{0.40}\right) = \Pr(-0.5 < Z < 0.5)$$

$$= \Pr(Z < 0.5) - \Pr(Z < -0.5) = 0.6914 - 0.3085 = 0.3829.$$

(b) The sample size is now $n = 100$, so $\mu_{\bar{X}} = \mu = 8$ and $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{100}} = 0.20$.

The z-score is now given by $Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} < \frac{\bar{X} - 8}{0.20}$.

Therefore,

$$\Pr(7.8 < \bar{X} < 8.2) = \Pr\left(\frac{7.8-8}{0.20} < Z < \frac{8.2-8}{0.20}\right) = \Pr(-1.0 < Z < 1.0)$$
$$= \Pr(Z < 1.0) - \Pr(Z < -1.0) = 0.8413 - 0.1587 = 0.6826.$$

D3. The Central Limit Theorem says that the sampling distribution of sample means approaches to a normal distribution $\mathrm{N}(\mu, \frac{\sigma}{\sqrt{n}})$ when the sample size gets large.

D4. (a)

Let X be the tuition of an undergraduate student. Then $\mu = \$4172$ and $\sigma = 525$.

$$\Pr(X < 4000) = \Pr\left(Z = \frac{X-\mu}{\sigma} < \frac{4000-4172}{525}\right) = \Pr(Z < -0.33) = 0.3707.$$

(b) $n = 36$, $\sigma_{\bar{X}} = \sigma/\sqrt{n} = 525/\sqrt{36} = 87.5$.

$$\Pr(\bar{X} < 4000) = \Pr\left(Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} < \frac{4000 - 4172}{87.5}\right) = \Pr(Z < -1.97) = 0.0244.$$

(c) The reason that the probability for part (b) is much lower than that in part (a) is because the sampling distribution of mean in part (b) has much smaller spread with a lot more values distributed near the centre than the population distribution in part (a). While few sample mean values, $\bar{X}$, are lower than 4000, there are many individual values, X, lower than 4000.

D5. $\mu = 1.5$ $\sigma = 0.5$ $n = 100$

$\Pr(\bar{x} > 1.1) = \Pr\left(z > \frac{1.0-1.5}{\frac{0.5}{\sqrt{100}}}\right)$

$= \Pr(z > -10) = 1$

The answer is A

D6. Assume that men's weighs are normally distributed…

$\mu = 172\ and\ \sigma = 29, n = 25$

$\Pr(150 < \bar{x} < 180) = \Pr\left(\frac{150-172}{29/\sqrt{25}} < Z < \frac{180-172}{29/\sqrt{25}}\right) = \Pr(-3.79 < Z < 1.38)$

Use table 1 and look up the area…Pr(Z<1.38) – Pr(Z<-3.79)=0.9162-0.0002=0.916
So, the probability is 0.916

D7. (a)

Let X be the credit card balance. Then $X \sim \mathrm{N}(\mu,\sigma)$ with $\mu = 2780$, $\sigma = 900$. So

$$\Pr(X < 2500) = \Pr\left(Z = \frac{X-\mu}{\sigma} < \frac{2500-2780}{900}\right) = \Pr(Z < -0.31) = 0.3783.$$

(b) Now we are looking at the distribution for the sample mean $\overline{X}$ in a sample of size $n = 25$. Then $\overline{X} \sim \mathrm{N}(\mu_{\overline{X}}, \sigma_{\overline{X}})$ where the mean and standard deviation for $\overline{X}$ are given by $\mu_{\overline{X}} = \mu = 2780$ and $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{900}{\sqrt{25}} = 180$. Therefore

$$\Pr(\overline{X} < 2500) = \Pr\left(Z = \frac{\overline{X}-\mu_{\overline{X}}}{\sigma_{\overline{X}}} < \frac{2500-2780}{180}\right) = \Pr(Z < -1.56) = 0.0594$$

D8.

Now we are looking at the distribution for the sample mean $\overline{X}$ in a sample of size $n = 10$. Then $\overline{X} \sim \mathrm{N}(\mu_{\overline{X}}, \sigma_{\overline{X}})$ where the mean and standard deviation for $\overline{X}$ are given by $\mu_{\overline{X}} = \mu = 625$ and $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{150}{\sqrt{10}} = 47.43$. Also, the total is given, so we must divide \$7000 by 10 to get a mean of 700.

Therefore,

$$\Pr(\overline{X} > 700) = \Pr\left(Z = \frac{\overline{X}-\mu_{\overline{X}}}{\sigma_{\overline{X}}} > \frac{700-625}{47.73}\right) = \Pr(Z > 1.58) = 1 - 0.9429 = 0.0571.$$

D9. This adjustment is called the FINITE CORRECTION FACTOR

When $\frac{\boldsymbol{n}}{\boldsymbol{N}} > 0.05, the\ following\ equation\ should\ be\ used$:

** Look for questions where the size of the population N is given

N=350

$\mu = 150$

$\sigma = 35$

$n = 50$

$\frac{\boldsymbol{n}}{\boldsymbol{N}} = \frac{\mathbf{50}}{\mathbf{350}} = \mathbf{0.142} > 0.05$

$\frac{\sigma}{\sqrt{n}} = 35/\sqrt{50}$=4.949747468

$$\sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{350-50}{350-1}} = 0.92714554$$

$$Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}} = \frac{160-150}{4.949747468(0.92714554)} = 2.18$$

Pr($\bar{x}$<160)=(Z<2.18)=0.9854

See you in January! Have a safe and happy holiday!